

<http://www.e-jemed.org/>

ISSN : 1298-0137

**e - JEMED**

The Electronic Journal  
of Evolutionary Modeling  
and  
Economic Dynamics

**Article number:** 1029

**Please cite this article as following:**

James P. Dow, Jr., 2003, Neighborhood effects and the distribution of income in cities, The Electronic Journal of Evolutionary Modeling and Economic Dynamics, n° 1029, <http://www.e-jemed.org/1029/index.php>

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## **Neighborhood effects and the distribution of income in cities**

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### **Abstract**

The aim of this paper is to explain patterns of household income within an urban area. The paper constructs a model of a linear city with inhabitants of different incomes and with properties of varying quality. Simulation of the model finds that population growth combined with neighborhood effects can produce a non-monotonic pattern of income. Sensitivity analysis shows how the outcome depends on the characteristics of the city. In particular, it is found that the speed of population growth is very significant. This is important as US cities, particularly in the West and South, have been characterized by rapid population growth.

**Keywords:** Residential Location, Urban Structure, Agent Based Economics.

**JEL:** R12, R14, C63

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## 1. Introduction

An important concern for urban economics is explaining the patterns of household income within an urban area. Because a common pattern in US cities is for a concentration of the poor in the inner city, the literature has tended to focus on neighborhood change resulting from higher-income families moving to the suburbs. Of course, the actual pattern of income distribution in urban areas is more complicated. Rather than income increasing monotonically as one moves towards the edge of urban areas, cities can be characterized as consisting of a variety of clusters of different income groups. A particular pattern seen in cities as varied as Los Angeles and New York is for both rich and poor to be located in desirably geographic areas towards the middle of the city, with groups of middle and higher income located farther out, and often with lower-income groups again located further out. The question is, if there are more desirable places to be, whether close into the city, or at the outsides of the city, why are they not exclusively occupied by higher-income groups?

This paper explores one answer to that question; that neighborhood externalities, sometimes called boundary effects, can result in a non-monotonic pattern of income distribution. It constructs a model of a linear city with inhabitants of different incomes and with properties of varying quality. Computationally, the paper is novel by its use of an agent-based approach. Instead of modeling the city by a set of equilibrium conditions, it determines the outcome according to the decisions of individual agents, using a simulated auction market that allocates households to properties dynamically.

The paper simulates the evolution of the city in response to an increase in population and shows how non-monotonic patterns of income can persist when there are neighborhood externalities and the city undergoes a growth spurt. The importance of the rate of growth is a key result of this paper. Fast-growing metropolitan areas are more likely to see an uneven income pattern than slow growing areas, even long after both areas have stopped growing. One of the advantages of the simulation approach taken in this paper is that it can conveniently incorporate these kinds of considerations into the analysis.

Los Angeles provides an example of this process. The Los Angeles area consists of a varied geography with significant differences in temperature and natural amenities (such as shoreline and mountains) and differential access to the commercial cores (Song, 1994, Small and Song, 1994, and Giuliano and Small, 1991, look at residential distribution and density in Los Angeles given the multiple commercial cores). While the most desirable areas (better climate, better accessibility) are generally higher income, and the least desirable areas, which tend to be at the outskirts of the region, are lower income, the overall pattern is quite varied. In particular,

there are some low-income neighborhoods in desirable geographic regions. In part, this seems to be a result of the rapid growth in population. When the city was small, low-income households located in the least desirable areas. As the population of the city increased, the new upper and middle income residents should have displaced the low-income households, forcing them farther out into even less desirable neighborhoods. This did not happen; the higher-income groups did not filter into the more desirable neighborhoods, but rather developed neighborhoods further out. We find both high-income and low-income households in the middle of the city and high-income and low-income households around the edges.

The traditional way of modeling the income distribution in an urban area is with a linear city with an employment center at one end. Given a population with heterogeneous income and preferences over consumption and location one can determine residential location patterns with respect to the employment center. Straszheim (1987) provides an overview of this literature. The basic result is that if the income elasticity of land or housing exceeds the income elasticity of the value of better access to the central city then income will increase as the population moves away from the central city. However, constructing an equilibrium model of a city with a population with heterogeneous income can be quite complex, as shown by Beckman (1969) and Montesano (1972) who model the effect of the distribution of income on location.

Of course, income elasticity is not the only factor that will affect location. Heterogeneous preferences can result in some rich households that care more about transport costs wanting to live closer to the central city. Also, if public transit is not uniformly available, households too poor to afford a car will tend to live near transit, generally in the central city. However, this paper will not incorporate these issues in order to better focus on the direct income effects.

The issue of income distribution has also arisen in the context of the shift of higher-income households out of the central city and into the suburbs. A standard approach to modeling this has been to assume two locations ("city" and "suburbs") and two populations (say, "rich" and "poor") and to determine if one population tends to live in one location. A number of reasons have been given for this pattern. One reason offered for this is that the filtering down of housing quality may cause the rich to move to the suburbs where the new housing is built, leaving the poor to live in the inner city where the depreciated housing lies (For example, Muth 1972, Smith 1972 and Sweeney 1974, along with Brueckner 1977, Fogarty 1982, and Phillips 1981, for an empirical discussion). Migration may also reflect households "voting-with-their-feet", as higher-income groups move to areas with better provision of public services. Also, different income, cultural or racial groups may impose externalities on each other which may cause groups to cluster in exclusive neighborhoods, whether in the urban center or out in the suburbs. It is these neighborhood externalities that are the focus of this paper.

Bailey (1959) and Schelling (1978) provide early discussions of how this effect might produce clustering in cities. Significantly, Schelling shows how individuals of two types, locating on a grid, with preferences about their nearby neighbors can produce endogenous neighborhood formation and segregation. Oates, Bowrey and Baumol (1979) provide a formal model tying income levels to neighborhood quality. Their city consists of two areas, inside the city and outside the city, with the evolution of the city described by two linear difference equations: As rich people leave the city, housing deteriorates, and this deterioration of housing causes the rich to leave the city. They show how these two forces can combine to cause the rich to concentrate in the suburbs. Kanemoto (1980) posits two types of agents (“rich” and “poor”) and a city divided into two parts (“city” and “suburb”). The rich have an aversion to living near the poor. He shows that equilibria can result with the city consisting of all poor, all rich or a mix of both. Bond and Coulson (1989) combines the filtering and border literature. They divide households into high-income and low-income and the city into a large number of neighborhoods, although these do not have any spatial component. Households prefer to live in neighborhoods with a larger fraction of high-income families. Houses age, and as they become lower quality, they pass to low-income households. The paper establishes when there can be multiple equilibria of the allocation of households to neighborhoods and when housing passes from high-income to low-income groups. Benabou (1993, 1996) shows how segregation can result from education externalities. If human capital production depends not only on direct inputs, but also on the human capital investment of other individuals in the same neighborhood, there is an incentive for high-skilled (and high-income) individuals to cluster in the same neighborhood. This can produce persistent urban segregation by income. As seen, most of these models (with the notable exception of Schelling, 1978) were designed to capture a bifurcated region split into city and suburbs, with the idea of duplicating the movement of the rich out of the city and into the suburbs. There is a need to extend these models to include multiple levels of income and multiple neighborhoods with a spatial component.

This paper develops a model of a linear city combined with neighborhood externalities in the spirit of the previous boundary models. The model extends this literature to incorporate multiple income groups and neighborhoods so as to capture non-monotonic patterns of income. To do this, the paper must take a somewhat different modeling approach. The model will consist of a city with many discrete or “atomistic” agents and will follow their movement around the city. This approach is sometimes called “Agent-Based Economics” (Tesfatsion, 2000) as it determines the outcome by focusing on the interactions of individual autonomous agents rather than by imposing equilibrium conditions.

The model will be structured as a linear city consisting of discrete lots. It is assumed that there is an attractive feature at one end of the city, so that the lots decrease in desirability as they

move away from that end. This feature could represent an employment center, so that greater distance produces increased commuting costs, or it could represent desirable natural features such as a shoreline. Each lot is of a fixed size so that there is no tradeoff between density and transit time and so, rents and surrounding neighborhoods being equal, households wish to live closer to this feature. There will be a sequential auction market to allocate properties to households. The population will consist of households having a range of incomes. Initially, half of the population, consisting of a variety of incomes, will move into the city. In the second period, the second half of the population will move into the city, initially locating in the more distant properties. The property market in the city will be simulated to see how the city evolves over time, particularly to see if the higher-income households filter to the more desirable neighborhoods, or whether an alternate higher-income neighborhood forms farther out. What is found is that the non-monotonic outcome can persist for a long period of time, but that the persistence will depend on the rate of city growth and the strength of the neighborhood effects.

## 2. The model

The model consists of a linear city divided into discrete lots and a fixed number of households. To describe the model, the paper will first go through the characteristics of the lots and households and then the dynamic auction mechanism for allocating lots to households

### 2.1 Properties and Households

The city will have 100 lots, indexed from 1 to 100. The value of property  $k$  ( $A_k$ ) is the sum of the value from the natural amenities ( $A_k^*$ ) and the neighborhood value ( $A_k^n$ ). The amenity value is given by,

$$A_k^* = \phi_0 \exp(-\phi_1 k) \quad (1)$$

Where  $\phi_0$  and  $\phi_1$  are parameters, and  $k$  is the index of the lot, so that lot 1 has the highest value of the amenity and lot 100 the lowest.

The final population of the city will consist of 100 households, also indexed from 1 to 100. The distribution of household incomes follow an exponential decay function,

$$Y_j = \theta_0 \exp(-\theta_1 j) \quad (2)$$

where  $Y_j$  is household income,  $\theta_0$  and  $\theta_l$  are parameters and  $j$  is the index of the household. Households get utility from the value of the land they inhabit ( $A$ ) and consumption ( $C$ ) according to the utility function,

$$U = C^\alpha A^{1-\alpha} \quad (3)$$

They are subject to the budget constraint,

$$Y = r + C \quad (4)$$

where  $r$  is the per-period rent they pay for the lot they occupy and where the price of the consumption good is normalized to 1. Utility can then be expressed as a function of land value and the rental rate,

$$U = (Y - r)^\alpha A^{1-\alpha} \quad (5)$$

The population will be split into two (or more) cohorts. The first cohort will be the initial inhabitants of the city while the second cohort will arrive in a later period. In the two-cohort version of the model, the first cohort will consist of odd-indexed households and the second cohort of even-indexed households. Splitting the population this way ensures that the two cohorts will have similar income distributions.

The income of nearby residents is assumed to affect the desirability of a property. The value of the property due to this externality is given by,

$$A_k^n = \frac{\mu}{2h} \left( \sum_{i=1}^h Y_{k+i} / \bar{Y} + \sum_{i=1}^h Y_{k-i} / \bar{Y} \right) \quad (6)$$

where  $k$  is the property affected,  $\bar{Y}$  is the average income level of the population and  $Y_i$  is the income of the household renting *property*  $i$ . It is assumed that households prefer to live in neighborhoods with *higher* average incomes rather than preferring to live next to people with *similar* incomes. This may reflect the better provision of local services, less crime, or externalities from nicer lots. It does not capture as well sorting by similar types. Racial and cultural sorting is clearly a feature of US cities but is often also highly correlated with income and so would be

similar to the effects captured here. The variable  $h$  measures the size of the relevant neighborhood. The neighborhood effect is scaled by  $h$  so that changes in  $h$  do not affect the relative importance of that effect (for a property surrounded by properties of the average income). The variable  $\mu$  determines how important neighborhood externalities are relative to the natural amenities. This specification was initially modified by including a decay parameter that reduced the importance of the externality as neighboring properties become more distant. This did not have a qualitative effect on the results and so was left off the final simulations. The externality also could have been structured as adding or subtracting from the value of the lot depending on whether nearby incomes were above or below average, but for the simulations, all that would matter is the relative value across properties.

The formula needs to be modified to incorporate the effect of unoccupied properties and properties at the edge of the city. Properties that have never been occupied are assumed to have the income effect of the minimum income. Properties that are currently unoccupied but were once occupied have the income effect of the income of the previous tenant. The value of properties near the edge of the city are calculated using values of hypothetical lots outside the city. Properties outside the city at the low end (next to lot 1) have income effects equal to the maximum income, so that this end remains a center of attraction. Lots outside the city at the high end (next to lot 100) have income values equal to the minimum income.

It is assumed that individuals have an opportunity to live outside the city. The property value of living outside the city is given by  $\bar{A}$  and the rental cost by  $\bar{r}$ , which is determined exogenously.  $\bar{A}$  equals an amenity level below the least valued property in the city. This was constructed by using a natural amenity value below the lowest value in the city combined with the assumption that it is surrounded by properties with incomes at the minimum level. It is also assumed that any property in the city has alternate uses that offer the same return as properties located outside the city. Because of this, properties will not accept any bids under  $\bar{r}$ . The value of  $\bar{r}$  was arbitrarily set to 20% of the minimum income level. Different values of this parameter were tried, and while it was found to affect the level of rents, it did not have a qualitative effect on the pattern of incomes.

## *2.2 The Auction Process*

A housing market can be modeled as either a simultaneous or sequential process, depending on the desired level of abstraction. In this model, the “state” of the city is not parsimonious; there are 100 unique properties and 100 unique households. The problem is how to allocate the households to properties. One approach would be the standard Walrasian

equilibrium; however, the computational costs of this approach are high given the large state space and the needed information for the households. Instead, I will model the allocation process as a continuing series of auctions and so represent the market by a sequential process. The model will be simulated over a number of turns, each turn consisting of three basic steps.

(1) Determine the lots available to be rented.

The set of available properties consists of properties unrented at the last round and newly available properties. Each property has a probability ( $\pi$ ) of becoming “available” each period which will be referred to as the turnover rate. This is the only random element in the model; the allocation of agents and all agent decisions are deterministic. When a property becomes available, the previous renter is returned to the pool of possible renters. This can either be thought of as the same household, or a new household with the same income, but in either case the income distribution of the overall population remains the same after turnover.

(2) Rank the properties in terms of desirability.

Since the utility functions of all individuals are the same, the available properties can be ordered in terms of desirability. Households are assumed to know the characteristics of each of the properties for sale. This assumption is fairly reasonable, as the Internet, “multiple listing services” and the use of buyer’s brokers have made this information widely available at low cost. The households are assumed to be myopic about neighborhood change, so that they take the household income associated with surrounding properties to be the current income and not the income they expect in the future. Rational expectations would probably play an important role in neighborhood change, by speeding the shift to the new equilibrium, and perhaps by opening up other equilibria; however, in this context they are difficult to handle and are not key for the results.

(3) Take bids on each property, starting with the most desirable.

The auction starts with the most desirable property. All households that are not currently renting make bids for the property at the same time, with the winner being the household offering the highest bid.<sup>1</sup> The rent paid by the winning household is equal to the second highest bid,

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<sup>1</sup> It was found that allowing all households to bid produced continual movement of households around lots, leaving too many properties unoccupied, and slowed convergence both with and without neighborhood



reflecting that in an auction, the household only has to just beat the next highest bidder. Once a household wins the auction, they are assumed settled and can no longer bid for other properties in that round. The process is repeated for each lot, by declining desirability. In the first round, this means starting with lot 1 and then proceeding in order. In subsequent rounds, desirability may not match the numeric order of the lots due to neighborhood effects.

The bid will be the highest rent a household is willing to offer given their opportunity cost of living outside the city. Increases in income result in higher bids as long as the alternate property outside the city is worse than the properties in the city. This is because increases in income increase the utility of households whether they live in the city or outside of the city. Since there is diminishing marginal utility from consumption of the good, higher income people will also want to consume higher quality property, which means a greater willingness to buy properties in the city. From the utility function, the bid function is given by,

$$r_{jk} = Y_j - (Y_j - \bar{r}) \left( \frac{\bar{A}}{A_k} \right)^{1-\alpha/\alpha} \quad (7)$$

where  $r_{jk}$  is the bid of the  $j^{\text{th}}$  household for the  $k^{\text{th}}$  property. Structuring the bid this way ignores strategic behavior by households with regards to two issues. Households do not take into account the behavior of other households that bid (although using the second highest bid reduces the importance of this). Perhaps more significantly, they do not take into account that other properties are available and will be auctioned off later. Starting the auction with the most desirable properties is designed to reduce the importance of this last issue and focuses the equilibrium on the highest income households residing on the most desirable properties.

### 2.3 The simulation and parameter values

The experiment will be to determine the outcome that will result in a growing city with neighborhood externalities. The exercise is qualitative in the sense that the goal is to establish when non-monotonic income distributions will develop. Since this is done by simulating the process described above, values for the parameters must be set. The base values for the parameters are given in Table 1.

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effects. An alternative specification subtracted a “cost of moving” from the household’s bid. The results were similar to the no-bidding-by-settled-renters assumption, and so the simpler specification was chosen.

Table 1. Parameter values

$A_{\max}^*$	1
$\pi$	0.1
$h$	5
$\phi_1$	1
$\phi_2$	0.003
$\mu$	See below
$\theta_1$	1
$\theta_2$	0.005
$\alpha$	0.5

For the purposes of the simulation, certain parameters, such as  $\alpha$ , are arbitrary and are chosen for convenience. Other parameters, such as those affecting turnover or neighborhood effects will be critical and so the next section will determine the importance of these parameters for the results. The simulation program was written in Matlab and is available from the author on request.

### 3. The effect of immigration into the city

The first experiment will determine the effect of immigration into the city on the pattern of income distribution. In the first period, all properties are available for lease. The first cohort (of 50 households) will bid for the properties. As will be seen, this will produce an initial allocation of households, with the richest households in the best properties and the poorest households in the worst. In the second period, the second cohort arrives in the city and bids for the remaining properties along with any other lots that open up. The simulation will last for 50 periods, with the distribution of the city in the 50<sup>th</sup> period compared with the monotonic distribution as a measure of the outcome. The first part of this section looks at the evolution of the income distribution in the city in the absence of neighborhood effects, then the experiment is repeated with strong neighborhood effects. To quantify the results, so as to evaluate their sensitivity to the parameter values, a measure of the deviation of the distribution from the monotonic pattern is constructed. The effect of changes in the strength and extent of the neighborhood effects and the importance of the natural amenity are evaluated.

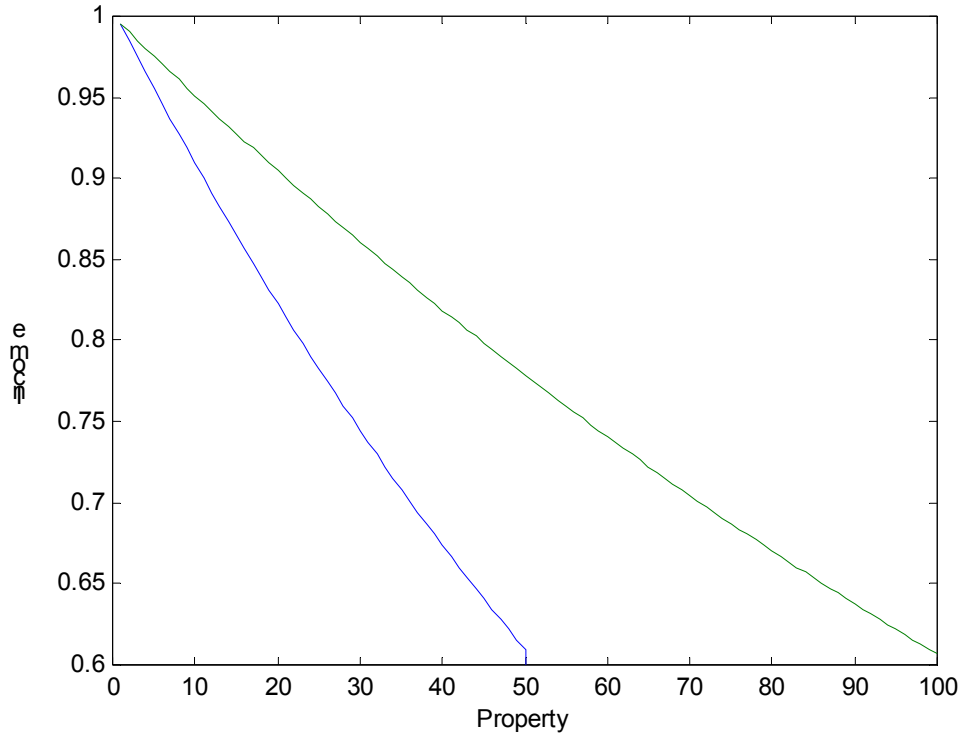


Figure 1a. Distribution of households after turn 1.

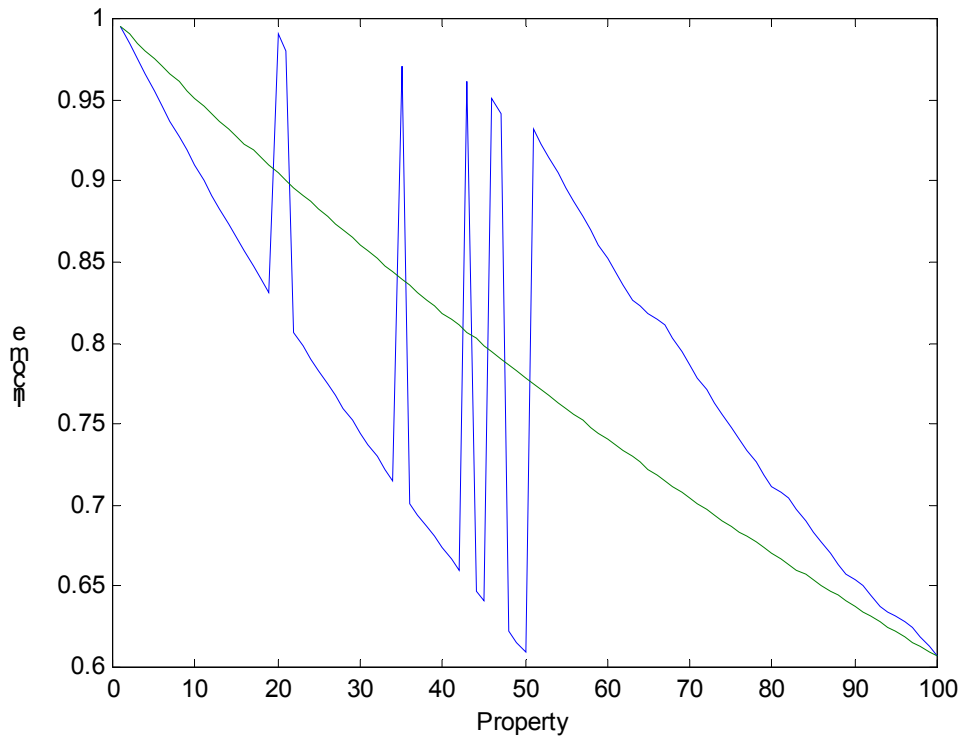


Figure 1b. Distribution of households after turn 2.

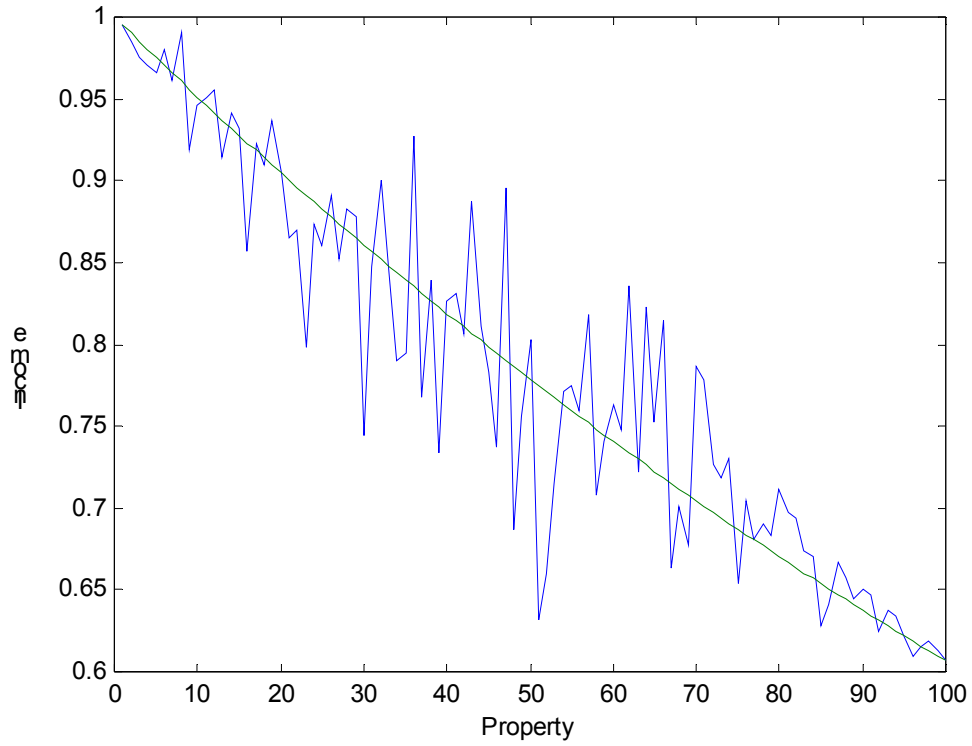


Figure 1c. The distribution of households after 25 turns

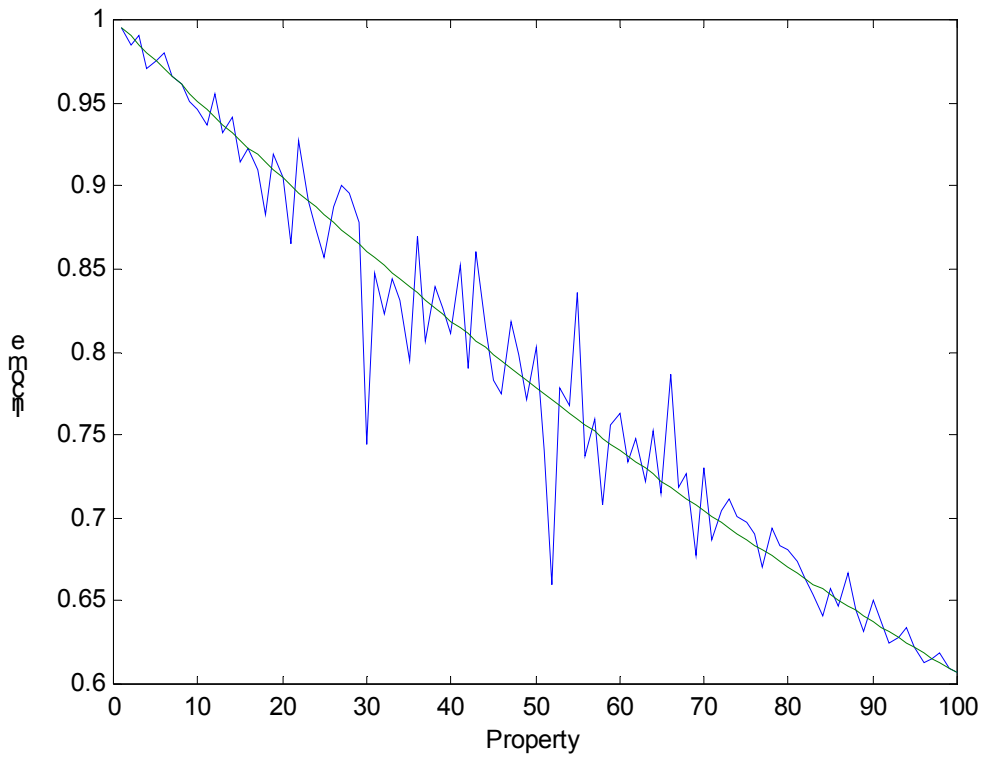


Figure 1d. The distribution of households after 50 turns

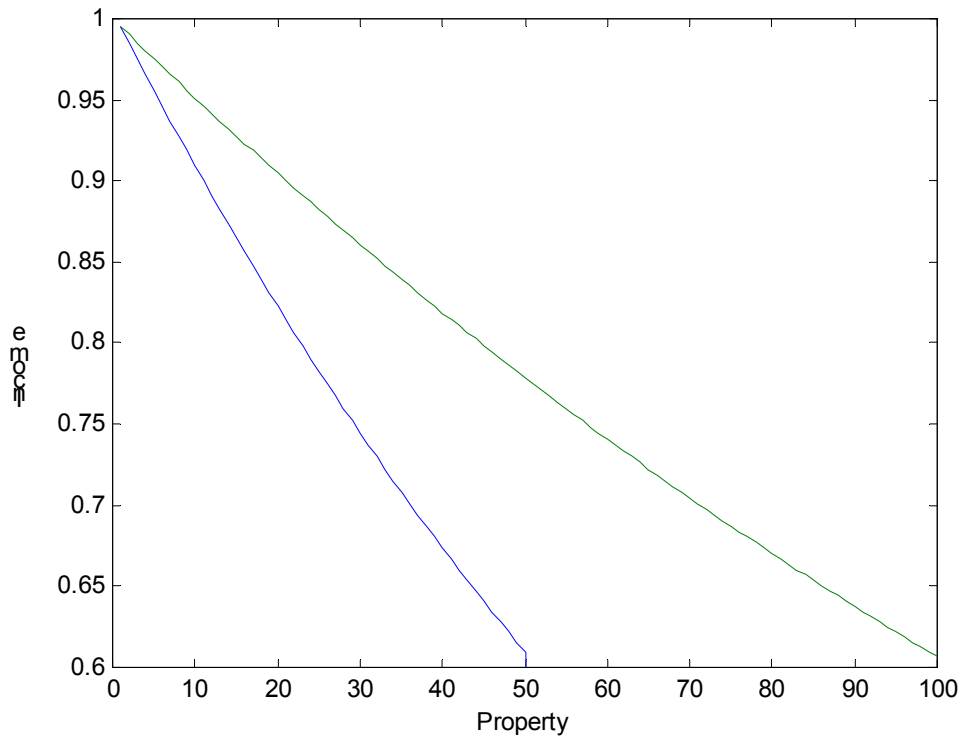


Figure 2a. The distribution of households after 1 turn – with neighborhood effects.

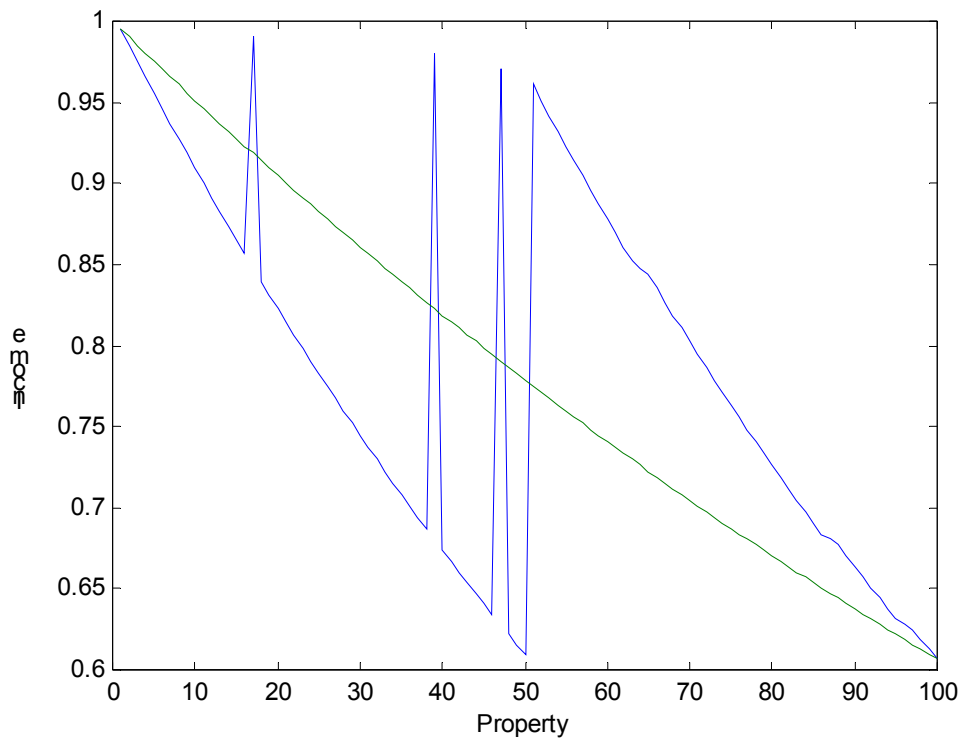


Figure 2b. The distribution of households after 2 turns – with neighborhood effects.

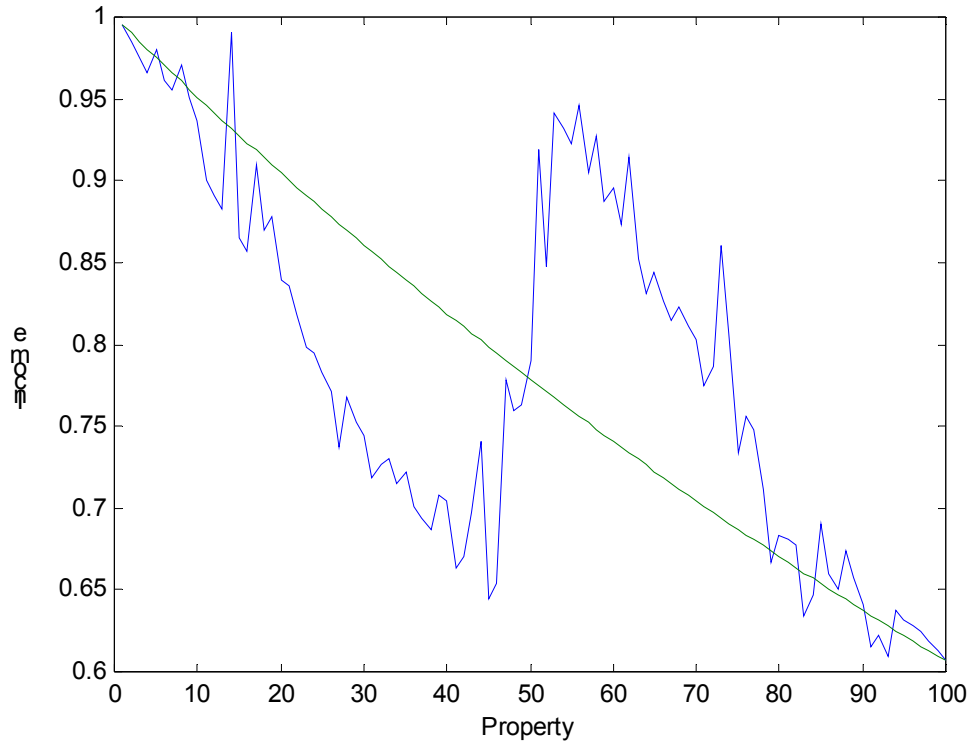


Figure 2c. The distribution of households after 25 turns – with neighborhood effects

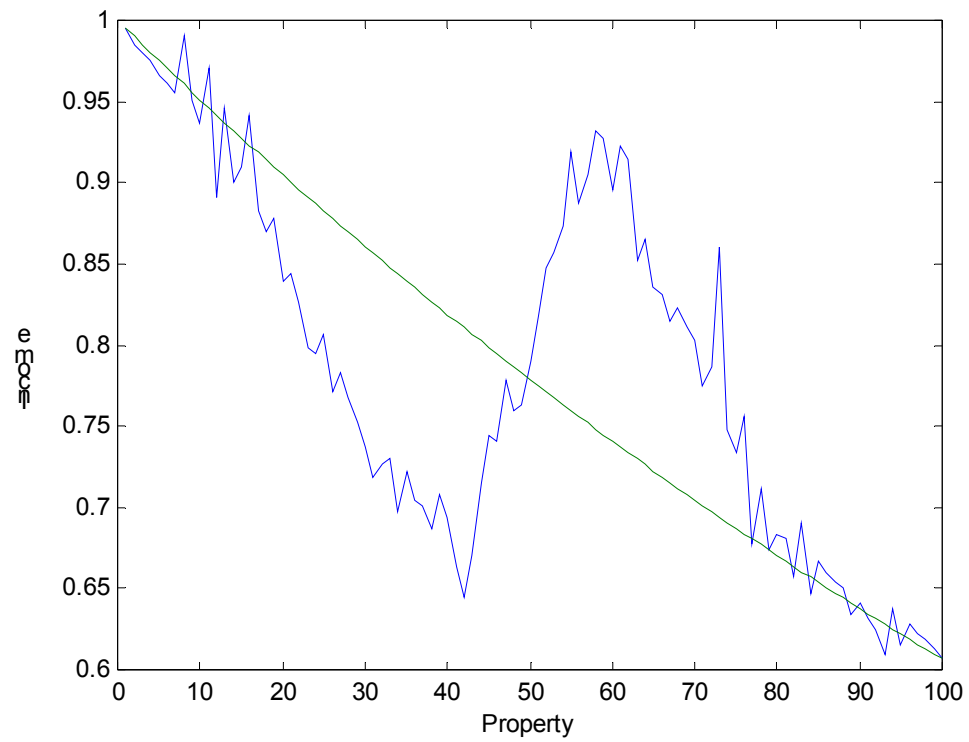


Figure 2d. The distribution of households after 50 turns - with neighborhood effects.

The behavior of the model in the absence of neighborhood effects is seen on figures 1a-d. This shows the pattern of development of the city for a representative simulation. Each plot shows the level of income of the household renting each property. For reference, a line is drawn showing the distribution of income if both cohorts were resident in the city and properties were allocated from highest income to lowest income. This will be referred to as the monotonic outcome. The property with the highest amenity value is located on the left end of the city. At the end of the first period and the first round of bidding (figure 1a), the households are located in the first 50 lots while the second 50 are unoccupied. There is a smooth income distribution with the highest-income household occupying the first (best) property and with the lower-income households holding the less desirable properties. The second period adds in 50 new households (figure 1b). Some of the first 50 properties will become available due to turnover, otherwise the new households will have to live in the second 50 properties. At the start of this period, the value of properties are still ordered from 1 to 100. Because of this, the properties in the range from 1-50 that are newly turned over will be held by the highest-income individuals in the market, with the rest of the households smoothly distributed over the remaining properties. At this point, the city has a jagged distribution of income by construction; some richer households must live towards the middle of the city as these are the properties that are available. The question is, will the distribution remain this way, or will the newly arrived rich filter to the left? Figures 1c and d show what happens after 25 periods and 50 periods respectively. As can be seen, the distribution of people collapses towards the ideal, although there is still some variation. In general, the initial influx of population will cause an uneven pattern of income at the start, but the effect will diminish over time.

In contrast, figures 2a-d show what happens when there are neighborhood externalities ( $\mu = 5.0$ ). Figure 2a shows the situation at the end of the first period, which is the same as figure 1a. In the second period, the new 50 households move into the city and bid for the available properties, which include the farthest 50 and any of the first 50 lots that turn over. It is important to point out that the households do not anticipate the effect of those moving in after them. The household renting property 51 believes it to be the best property of those from 51 to 100, although it is near the low-income household in property 50. *Ex post*, properties further out will seem better, as they are more centered in higher income areas. Unlike the previous example, the second peak does not disappear over time, as can be seen in figures 2c and 2d. Indeed, the peak shifts slightly to the right, moving between the two low income areas. This demonstrates how neighborhood externalities can cause the kind of income patterns discussed at the start of the paper. The best properties are held by high-income individuals and the very worst properties are held by the lower-income individuals. However, one can get reverses, with higher-income

households locating farther out and in less-desirable properties as they form high-income neighborhoods when they first move in. Indeed, middle-income individuals in the older part of the city (lots 1-50) may prefer to move out to the second high-income neighborhood.

In order to examine this phenomena in more detail, a measure quantifying the difference between figures 1d and 2d is needed. To do this, the paper will use the sum of the squared deviations from the monotonic pattern at period 50 as a measure of convergence. The smaller this number, the more the new population has filtered through and the less important is the second peak. To determine the average behavior of the model, 400 simulations of the kind just examined will be run, and the average squared deviation across the simulations will be calculated. This will be referred to as the “average deviation”. It is really a double average; first, the average squared deviation across the city, and then the average of this average over the simulations. To determine the range of results across the simulations, the standard deviation of this average deviation will also be calculated.

Figure 3 plots the average deviation for values of  $\mu$  from 0 to 9.0. To get an idea of the spread of outcomes, two additional lines were added which show the average deviation plus and minus one standard deviation. As can be seen, the more important the effect of neighborhood externalities, the larger the average deviation. The size of the deviation increases quite rapidly until about  $\mu = 5$  and then levels off.

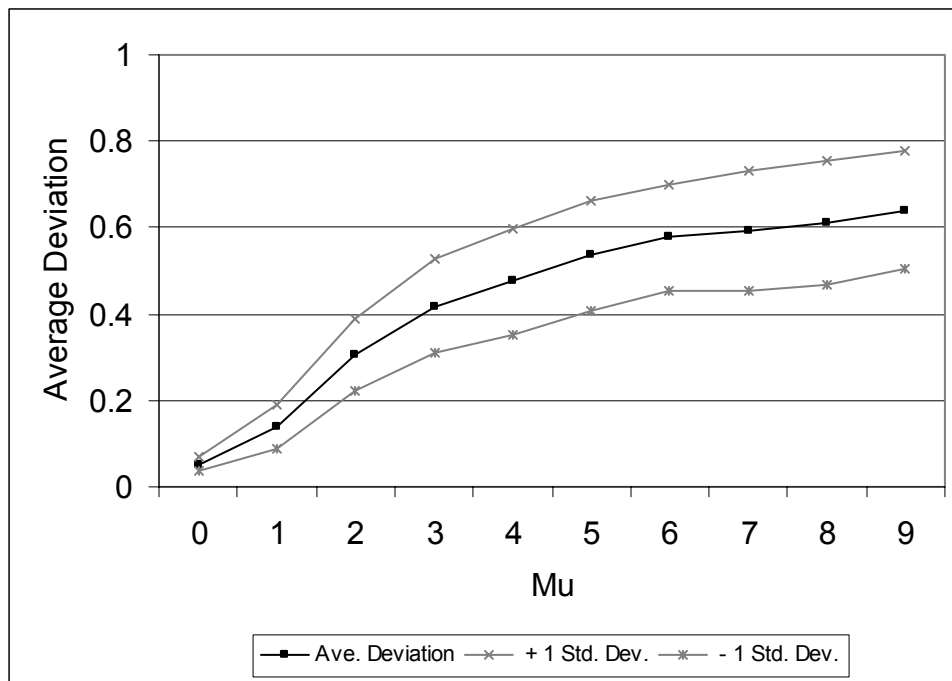


Figure 3. The strength of neighborhood effects and the average deviation.



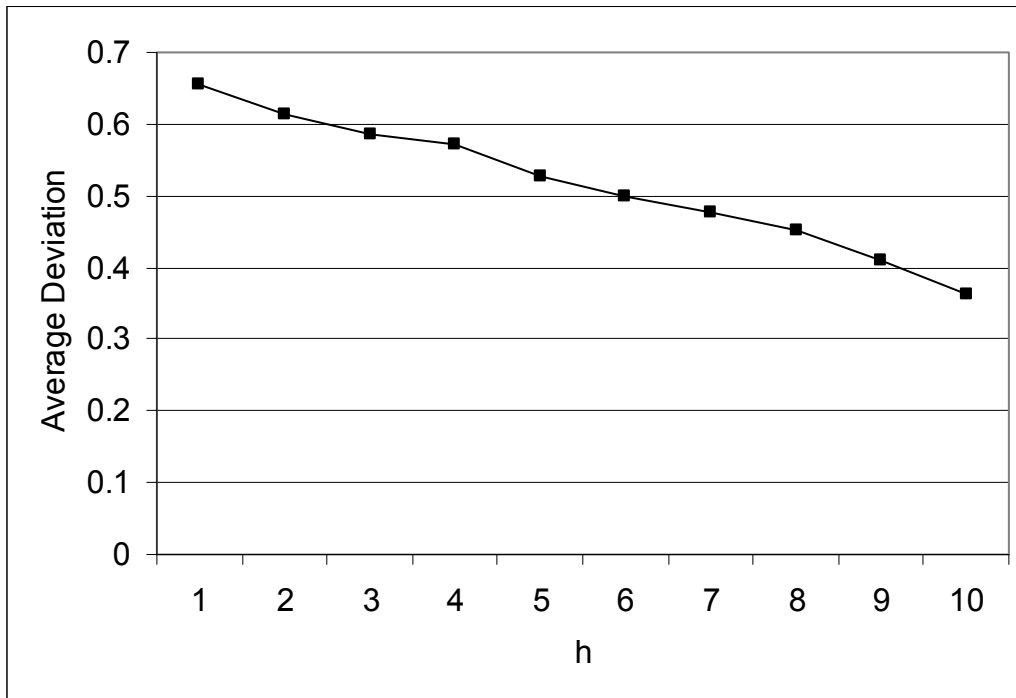


Figure 4. The effect of  $h$  on the average deviation.

The effect of the neighborhood externality can also be altered by changing the value of  $h$ . As  $h$  gets larger, the particular neighborhood starts to matter less, which reduces the importance of the neighborhood externality. This can be seen on figure 4, which plots the average deviation as  $h$  goes from 1 to 10.

The rate of turnover should also have an effect on the evolution of the city; however, with neighborhood externalities, the effect is complex. To take a hypothetical example, if there were no neighborhood externalities, and all properties turned over, the result would be the monotonic pattern of income, since the auction process is designed to produce this result. With neighborhood externalities, this probably would not be the result, since the previous incomes of the properties would still matter and so an uneven pattern might persist. To move towards the monotonic outcome, a specific set of vacancies would be needed. If there is a turnover of a relatively less-rich property in the first (better) rich neighborhood and of a higher-income property in the second rich neighborhood, the two households would tend to switch places, with the higher-income person moving to the better neighborhood. This changes the distribution towards the monotonic pattern. The more properties that are turned over, the more likely these kinds of exchanges would be made, and so the quicker the transition to the monotonic pattern.

Table 2 reports the results of simulations with varying levels of turnovers. As turnover increases, the average deviation does fall. Indeed, when turnover becomes high enough, the city behaves just as if there were no neighborhood effects.

Table 2. Turnover

Turnover rate ( $\pi$ )	Average Deviation (Standard Deviation)
0.05	0.9053 (0.1114)
0.1	0.5298 (0.1270)
0.2	0.1744 (0.0750)
0.3	0.0288 (0.0332)

Increasing the value of  $\phi$  (the speed of the decline in the value of the natural amenity) increases the importance of *distinctions* between properties in terms of the natural amenities. The sharper these distinctions are, the more the rich will have an incentive to locate closer to the left side, and ignore neighborhood effects. Table 3 reports the values of simulations with increasing values of  $\phi$ . As  $\phi$  gets larger, the average deviation gets smaller, and the distribution of income more closely matches the monotonic pattern.

Table 3. Change in the amenity

Decline in amenity ( $\phi$ )	Average Deviation (Standard Deviation)
0.001	0.6675 (0.1461)
0.002	0.5926 (0.1367)
0.003	0.5298 (0.1270)
0.004	0.4769 (0.1214)
0.005	0.4339 (0.1160)

Overall, a persistent and pronounced deviation from the monotonic income pattern requires significant neighborhood effects along with turnover that is not too high and with the geographic range of the neighborhood effects not being too large.

#### 4. Slow growth

The reason that there is an uneven pattern of income in the city is that there are enough high-income households put together at one time to form a separate high-income neighborhood. This suggests that the pace of immigration may affect the way cities are structured. If the new residents were not added in all at once, but rather arrived in smaller groups, they might be able to disperse according to income and a second neighborhood might never form.

To evaluate this hypotheses, the second cohort was split into 10 separate sub-cohorts and added sequentially to the city. The sub-cohorts were produced as before, allocating individuals to the different groups successively so that each sub-cohort had a similar income distribution to the initial cohort. What was found was that higher-income households of the new cohort were able to find housing in the better part of the city, so that the outskirts of the city remained relatively low income. As each new group arrived in the city, this process continued, and the city converged on the monotonic pattern.

Table 4 shows the average deviations of the earlier experiments (with  $\mu=0$  and  $\mu=5.0$ , from figure 3) and the slow growth experiment with  $\mu = 5.0$ . Slow growth makes the city look as if there are no neighborhood effects, even though the effects are quite strong.

Table 4. The effect of slow growth

	Fast Growth		Slow Growth
	$\mu = 0$	$\mu=5.0$	$\mu=5.0$
Average Deviation (Standard Deviation)	0.0525 (0.0168)	0.5349 (0.1254)	0.1061 (0.0602)

This experiment illustrates one of the advantages of the simulation approach. The model can easily incorporate dynamic factors, such as slow or fast growth, in a way that would be difficult for analytical models.

## 5. Conclusion

The income patterns of American cities are quite complicated. There are high- and low-income households in seemingly desirable locations, such as near employment centers and or near natural amenities, and there are high- and low- income households in less desirable locations. This paper has shown one way that these patterns can arise in cities characterized by neighborhood externalities and fast growth.

The welfare implications of this outcome are not clear. In this model, the poorer households gain in terms of location when the rich locate farther out, rather than when they are displaced by the new households and the monotonic pattern occurs. This is because the poor will tend to live in poor neighborhoods in any case. In the two-peaked equilibrium, one of the poor neighborhoods is located closer in to the amenities, and so is better off. To the extent that there is mixing of the poor and the rich, the poor have the benefit of richer neighbors. Benabou (1996, 1993) argues that this can be important, particularly when education depends on local funding or other inhabitants of the neighborhood.

Because some of the rich are worse off in terms of location in the two-peaked equilibrium there are private incentives to break it; however, this requires a high degree of coordination to shift enough properties from low-income to high-income. Once this did happen, the process would be self-reinforcing and “gentrification” could quickly happen. This suggests that these income patterns might be unstable equilibria, and open to continued evolution in the future.

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