http://www.e-jemed.org/

ISSN : 1298-0137

e - JEMED
The Electronic Journal of Evolutionary Modeling and Economic Dynamics

**Article number:** 1020

**Please cite this article as following:**

Agnès d'ARTIGUES and Thierry VIGNOLO (2002), Title, The Electronic Journal of Evolutionary Modeling and Economic Dynamics, n° 1020, http://www.e-jemed.org/1020/index.php

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# *Long-run Equilibria in the Monetary Policy Game*

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This paper presents an evolutionary interpretation of the Barro-Gordon monetary policy game in order to capture strategic behaviors explaining the convergence to low inflation rates. The model assumes the government as well as the private sector are boundedly rational players. The behavioral rule of the government is imitation of the best player whereas the public decision follows adaptive expectations. In this evolutionary monetary policy game, we find that the Pareto-efficient outcome can be observed as the long-run equilibrium of the game when inertia is included in the model. Then, we question the relevance of the model when the public can make rational expectations. It turns out that previous results are not modified, meaning that the result in the Barro-Gordon model further relies on the perfect rationality given to the government rather than on the rational expectations.

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# LONG-RUN EQUILIBRIA IN THE MONETARY POLICY GAME

**Agnès d'ARTIGUES*** **and Thierry VIGNOLO** †

18th October 2002

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# 1 Introduction

A simple observation of the evolution of inflation rates of many industrialized countries during the last thirty years draws two main periods. The first one was characterized by high and differing inflation rates whereas the second one, from the middle eighties to recently, has seen most advanced countries reach low inflation rates.

The high inflation period has been considered in the literature through the monetary policy game defined by Kydland and Prescott (1977) and made popular by Barro and Gordon (1983). These authors have shown that policy makers face a time inconsistency problem under discretionary policies, namely an inflationary bias results from attempts to achieve a level of output higher than the natural rate.

The literature on optimal monetary policy has proposed institutional arrangements for central banks in order to avoid the inflationary bias[1], as the delegation of the monetary policy to a conservative central banker. This solution, proposed by Rogoff (1985), consists of appointing someone who puts less weight on employment stabilization than the government. The Rogoff solution has become the major argument supporting central bank independence, which is by now the most often recommended mechanism to achieve price stability[2]. As a result, we have witnessed the establishment of independent central banks in many of industrialized countries, and even beyond in some transition economies, with relative success regarding the level of inflation[3].

The convergence to low inflation rates operated during the past fifteen years has not always been accompanied by central bank independence. Some countries have chosen alternative instruments, such as inflation targets, fixed exchange rates, and inflation contracts, that may help to reach low and stable rates of inflation[4]. Moreover, it is possible to find cases of countries in Europe and elsewhere, choosing central bank independence, that started to converge to low inflation before opting for the independence solution. Hayo and Hefeker (2001) show that central bank independence is neither necessary nor sufficient and that it does not ensure price stability in all cases. Thus, central bank independence seems to have played a minor part in the convergence to low inflation rates observed recently.

The present paper represents an attempt to explain at once the inflationary bias and the convergence to low inflation rates in the monetary policy game, without resorting to the "conservative" solution or other institutional arrangements. It is necessary to leave the monetary independence issue by arguing that countries have followed a convergence process with a form of loose coordination of monetary policy. It is not a discretionary solution which can explain the convergence to low inflation rates, but rather the common belief within industrialized countries on the benefits of low inflation[5]. We argue that there are evolutionary forces like "imitation of the best player" which have driven countries to the common belief in the benefits of low

[1]See Rogoff (1985), Cukierman (1987), Lohmann (1992), Persson and Tabellini (1993), and Walsh (1995).

[2]It must be noted however that conservatism and central bank independence are differing concepts. For more details on this point see Hayo and Hefeker (2001)

[3]See Hayo and Hefeker (2001), Hillman (1987). See also Blinder (1999) for a survey on central bank credibility around the world.

[4]New Zealand, Sweden, the United Kingdom, Switzerland, Australia, Israël, and Canada have assigned an inflation target to their central bank, which often have been made independent.

[5]The common belief about the benefits of low inflation rates could be compared to the past belief in a long-run inflation-unemployment trade-offs during the postwar fiat-money era. See McCallum (1995).

inflation .

The model we present is inspired by evolutionary game theory[6]. The structure of the Barro-Gordon original game is preserved, meaning both that the government realizes a short-run benefit from surprise inflation and that the public best strategy is to correctly anticipate the inflation rate. Departure concerns assumptions about the rationality of both the government and the public, namely the way of choosing the strategy to implement. In contrast to the Barro-Gordon setup, players are boundedly rational agents using the past experience and simple behavioral rules in order to elaborate a strategy. Some degrees of inertia are included in the behavioral rules representing both price stickiness and the importance attributed by the government to price stability. The main innovation concerns the government possibility to learn by way of an imitation process what is the best strategy in the long-run. Evolutionary game theory makes it possible to include such a learning process in the game by considering several monetary policy games played simultaneously and over time, namely a multi-country setup.

The model shows how a low-inflation equilibrium can be reached in the long-run. This result is firstly obtained under adaptive expectations on the part of the public, when some rigidities or inertia are included in the learning process. It reflects that the resolution of governments to follow a strategy in the long-run has played a crucial part for the convergence to low inflation rates. It can also be related to the literature considering price stability as a social choice, i.e., mainly determined by the importance being attached to fight inflation. We then question the relevance of the model when the public can make rational expectations. It turns out that previous results are not modified, meaning that the result in the Barro-Gordon model further relies on the perfect rationality given to the government (i.e., the government maximizes an objective function expressing the aggregated and correctly revealed individual preferences) rather than on the assumption of rational expectations.

The rest of this paper is organized as follows. Section 2.1 presents the Barro-Gordon policy game outlining the main assumptions of the model concerning the rationality and information level attributed to the players. The evolutionary model is described in Section 2.2. Results and extensions are given in Sections 3 and 4. Section 5 concludes by a discussion related to a possible macroeconomic interpretation of the model.

# 2 The monetary policy game

## 2.1 The Barro-Gordon setup

Barro and Gordon (1983) consider a repeated interaction between two players, a government or a central bank (say player *i*) and a private sector (say player *j*). In this game, inflation and output are related via a surprise-supply function,

$$y = y_n + (\pi - \pi^e), \tag{1}$$

where $y$ is output, $y_n$ is the natural rate and $\pi^e$ denotes expected inflation. The

[6]For book-length introductions to evolutionary game theory, see Vega-Redondo (1996) and Weibull (1987).

government, which dislikes inflation but desires output, controls the inflation rate $\pi$ via monetary policy. Its single-period welfare function is given by

$$u_i = a(\pi - \pi^e) - \pi^2, \ a > 0 \tag{2}$$

that is, the government prefers output to be above that determined by the private sector. The government is assumed to choose directly the inflation rate. Remark that the priority the government gives to price stability or surprise inflation is resumed in (2) by constant $a$.

The private sector, on the other hand, prefers to correctly anticipate the inflation rate, in order to avoid inflation surprise from the government. The single-period welfare of the public, given by

$$u_j = -(\pi - \pi^e)^2, \tag{3}$$

formalizes such an idea. The private sector is assumed to choose $\pi^e$, the expected inflation rate[7]. All departures from the expected rate decrease the private sector utility.

Let the strategy space be restricted to $\pi = (0, 1)$ (and $\pi^e = (0, 1)$), and denote $L = 0$ and $H = 1$ respectively the low and high inflation rate. If, for convenience, we set $a = 2$, the interactions between the government and the private sector can be summarized in the payoffs matrix

| | $\pi^e = L$ | $\pi^e = H$ |
|---|---|---|
| $\pi = L$ | $0, 0$ | $-2, -1$ |
| $\pi = H$ | $1, -1$ | $-1, 0$ |

In this game, $\pi = H$ is a dominant strategy for the government, that is the high inflation choice realizes the best gain whatever the public strategy. On the other hand, the best reply for the public is to always correctly anticipate the inflation rate selected by the government. In the Barro-Gordon setup, the private sector knows perfectly that $\pi = H$ is a dominant strategy, so that the only Nash equilibrium of the game is $(H, H)$.[8]

However, the Nash equilibrium $(H, H)$ is Pareto inferior to the low-inflation solution $\pi = \pi^e = L$. This optimal policy, both from the standpoint of the government and the public, is usually referred to the *ideal policy* but does not constitute a Nash equilibrium. Barro and Gordon (1983) consider the infinite horizon of the game, assuming that the private sector uses "punishment srategies", and evaluate the capacity of reputational forces to sustain levels of inflation lower than the one-period Nash solution[9].

[7]It is implicitly assumed that the public, although composed by a large number of agents, reacts as a single player. This assumption raises the coordination problem of the private sector, i.e., how can the public coordinate on a strategy ? On the subject, see al-Nowaihi and Levine (1994).

[8]In order to achieve this equilibrium, it is necessary to assume that not only the payoffs are common knowledge but also the players rationality, i.e., the way each player chooses a strategy from the information at his disposal.

[9]Remark that considering the finite horizon of the game, in a *common knowledge* situation, leads to the one-period Nash solution since this issue is the only perfect equilibrium in the sense of Selten (1975). See also Selten (1978).

They find that the ideal policy (the Pareto-dominant equilibrium) cannot be enforced by the only potential loss of reputation, when the game is repeated. Further, infinite horizon games, as mentioned by Backus and Driffill (1985), generate multiple Nash equilibria without any rule permitting the selection of one particular solution[10].

In the Barro-Gordon analysis of the monetary policy game, the structure of the game and the players rationality are *common knowledge*, meaning that each player knows the full structure of the model, including all objective functions. As a result, the only Nash equilibrium of the game is the one in which the government is tempted to inflate and the public correctly anticipates this choice. In our view, more satisfactory is to consider that lack of information leads players to behave less rationally than the usual notion of perfect rationality. We do this in the next section analyzing the Barro-Gordon game with the evolutionary game theory.

## 2.2 An evolutionary setup

In contrast to the Barro-Gordon setup, evolutionary game theory assumes players are boundedly rational agents using the past experience and simple behavioral rules in order to elaborate a strategy. The model thus includes learning and dynamic considerations in the analysis. The game and the players rationality are not *common knowledge*. This means that each player ignores its opponent's payoffs and the way the latter takes its decisions. We use methods introduced by Kandori, Mailath and Rob (1993) and Young (1993) to examine the long-run behavior of players in the monetary policy game.

Consider a set of $n$ private sectors (each of them located in a country) denoted by $J = \{1, ..., j, ..., n\}$. Let $I_T = \{1, ..., i, ..., n\}$ be a set of $n$ governments for $T$ periods, $T$ a finite integer. Time is measured discretely and indexed by $t = 1, 2, 3....$ At period $t$, each private sector $j \in J$ is randomly paired with a government $i \in I$ to play the game described by the payoff matrix of Section 2.1. Each matching remains fixed for a discrete interval of length $T > 1$[11]. After this interval of time, the matching process is reconsidered, each private sector being again randomly matched with an another set of governments. We note $\{i, j\}^T$ the couple formed by players $i$ and $j$ for $T$ periods. As we assume that players are randomly matched, the probability two given players meet is identical whatever these two players[12].

We define the evolutionary dynamics which describes the strategic behavior adopted by each player $i \in I$ and $j \in J$ in the long run. Let $g_t = (g_{1t}, ..., g_{it}, ..., g_{nt})$

[10]Backus and Driffill (1985) extend the analysis of reputation in the Barro-Gordon model by treating policy as a finite dynamic game and applying the Kreps and Wilson (1982) concept of *sequential equilibrium*. They find that inflation at the end of the term is the rational response of a government that cares about employment. It works because the public is uncertain about the true type of government. Even if the government announces its intention to fight inflation regardless of the output cost, the public is uncertain about whether it is simply an attempt to manipulate their expectations.

[11]The interaction between a government and a public describes a finite horizon game.

[12]We shall see below that players are myopic agents trying not to influence or anticipate the future behavior of their opponents. The myopic assumption is then partly justified in presence of random matching since the incentive to try to alter the future play of opponents is in this case small enough to be negligible.

be the strategy-profile of governments at period $t$, where $g_{it} \in \{L, H\}$ is the strategy of the government $i \in I$. In the same way, denote by $p_t = (p_{1t}, ..., p_{it}, ..., p_{nt})$ the strategy-profile of private sectors at period $t$.

The state space of the dynamic process defined below is identified with the set

$$\Omega = \{z = (z^I, z^J) : 0 \leq z^I \leq n,\ 0 \leq z^J \leq n\}, \tag{4}$$

where, at each $t$, $z_t = (z_t^I, z_t^J)$ is interpreted as the number of players $z_t^I$ and $z_t^J$ using strategy $L$ respectively in the sets $I$ and $J$. For convenience, states $z' = (n, n)$ and $z'' = (0, 0)$ will be directly written $z' = (L, L)$ and $z'' = (H, H)$. The evolution of state $z_t$ is realized via two different mechanisms: selection and mutation. The next two sections describe these mechanisms.

### 2.2.1 The selection mechanism

The selection component of the dynamics consists of a learning process where players are myopic and adaptive agents[13]. They do not form expectations about the future course of play and simply take into account the decisions made in the past to determine their strategies.

We assume that the learning process of governments is imitation whereas private sectors follow a best reply dynamics. One justification for this difference relies on the information received and considered by each set of players. We argue that the information available to private sector $j$ only concerns the strategy of government $i$, whereas the latter also considers the strategies used in the past by other governments (including foreign governments). This results from the different dynamics attributed to government and public players.

If one restricts the information considered at $t$ by each player $j$ to $g_{it}$ (the last strategy of government $i$ with whom $j$ is paired), then the only way for $j$ to behave is to play a best reply to $g_{it}$ at $t+1$.[14]

Let $t = t^*$ be the first period of $\{i, j\}^T$. In period $t \neq t^*$, each player $j \in J$ uses information on player $i$'s strategy at $t-1$ to compute a myopic best response, that is player $j$ chooses a strategy $p_j$ that satisfies the behavioral rule

$$p_{jt} \in argmax_{p_j}\ u_j(p_j, g_{it-1}),\ \text{if}\ t \neq t^*. \tag{5}$$

Due to the random matching assumption, player $j$ has no information about with whom he will be paired when $t = t^*$. Then, player $i$ considers only the fraction of the governments playing each strategy. In this case, the expected payoff function $U_j$ is defined as follows:

$$U_{jt}(p_j, g_{t-1}) = \sum_{i \in I} q_{ji} u_j(p_{jt}, g_{it-1}),\ \text{if}\ \ t = t^*,$$

[13]Section 4 allows the model to consider partly rational expectations, namely expectations which are halfway between rational and adaptive expectations.

[14]Remark that this assumption is similar to that of the retrospective or "naive" voters, whose decisions are made on the basis of a retrospective evaluation of macroeconomic performance. We shall see in Section 4 that adaptive and rational expectations are equivalent in our model.

where $q_{ji}$ is player $j$'s probability to encounter player $i$. As the probability that two given players meet is identical, $q_{ji} = 1/n$ for all $i \in I$ and $j \in J$.

The learning process of governments is driven by imitation. This means that changing from one strategy to another is dictated by such considerations as: How well do I perform compared to the other players $i \in I$? What is the strategy used by the most successful player? We think that imitation dynamics is a rule appropriate to the decision problem of governments, in which a strategy appears to be a complicated object depending on various contingencies. In order to simplify the matter, each government observes the past payoffs and the corresponding strategy of the other governments. The strategy giving the highest profit is then imitated. This learning procedure is called "imitate the best player"[15].

Let $u_{g_t} = (u_{g_{1t}}, ..., u_{g_{it}}, ..., u_{g_{nt}})$ be the payoff profile at $t$, where $u_{g_{it}}$ is the realized payoff of the strategy $g_{it}$. At $t+1$, each government finds the maximal payoff in $u_{g_t}$ and then imitates the corresponding strategy. That is, player $i$ chooses $g_{kt} \in g_t$ such that

$$k \in argmax_{\, l \in I} \; \{u_{g_l}\}. \tag{6}$$

We do not consider that all players simultaneously adjust their strategy at each period $t$. Rather, we assume that there is some inertia[16] in the learning processes (5) and (6). On the part of the public, macroeconomic justifications of inertia are related to price stickiness and nominal wage rigidity. The presence of inertia in the government learning expresses the importance attributed by the government to price stability, a high degree of inertia meaning the government preferences is to keep its strategy for a long enough period (perhaps in order to stick to its plans or to permit the strategy to reach its optimal profit).

Formally, each player independently with some fixed probability $\theta \in (0,1]$ receives the opportunity to update his strategy in each period. We denote by $\theta_i$ and $\theta_j$ respectively players $i$ and $j$'s opportunities of learning. The degrees of inertia in (5) and (6) are then expressed via the complementary probabilities $1-\theta_i$ and $1-\theta_j$.

**Proposition 1** *The states $z' = (L, L)$ and $z'' = (H, H)$ are the only stationary states of the monetary policy game.*

**Proof.** Assume the state is in state $z''' \in \Omega$, $z'''$ different from $z'$ and z". Then both strategies H and L are played. In this case, governments are leaded by the imitative dynamics to the strategy of the most successful player in $I_T$, which can be $H$ or $L$ depending on the opponent strategy confronted. As the public best reply to each of the two government strategies is unique ($H$ if $H$ and $L$ if $L$), the system yields $z' = (L, L)$ or $z'' = (H, H)$. ■

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[15]Another common learning process consists in imitating the best strategy. We believe that "imitate the best player" is a better rule in our setting as it requires less information and less ability to compute. Actually, it is only necessary that player $i$ observes the best player, without computing the average payoff of each observed strategy.

[16]The inertia assumption is a good justification of myopic behavior: as players know that only a small segment of agents changes its actions, strategies that proved to be effective today are likely to remain effective in the near future.

Proposition 1 means that, without perturbations, once the learning process (5)-(6) has reached one of the two states $z'$ or $z''$, it remains forever. The fact that $z' = (L, L)$ appears to be a stationary state in our model depends on the imitative rule followed by governments. Without experimentations of new strategies (or mutations), governments are imitating the only strategy present in the game. The next section introduces the possibility of experimentation or mutation in the learning process, in order to evaluate the long-run equilibria of the model in presence of perturbations.

#### 2.2.2 The mutation mechanism

Besides the selection mechanism, we assume that with a small probability $\varepsilon$, each agent plays an arbitrary strategy. The learning process is then perturbed by mutation, a phenomenon which may be interpreted as experimentation of nonoptimal strategies in the sense of (5)-(6) or the entry of a new player who knows nothing about the game. These mutations are independent across players and over time and occur after the learning adjustment.

The dynamic process (5)-(6) and the mutation mechanism together generate a Markov chain over the finite state space $\Omega$. Let $P(\varepsilon) = (p_{zz'}(\varepsilon))$ be the transition matrix over $\Omega$. The element $p_{zz'}(\varepsilon)$ represents $Prob\{z_{t+1} = z' \mid z_t = z\}$, that is the transition probability between states $z$ and $z'$. For $\varepsilon = 0$, $P(0)$ describes the *unperturbed process* which corresponds to the dynamic process (5)-(6). On the other hand, the matrix transition $P(\varepsilon)$ generates the *perturbed process* which includes mutations in (5)-(6) for $\varepsilon > 0$.

Let $\Delta^{|Z|}$ be the $\mid Z \mid -1$ dimensional simplex, where $\mid Z \mid$ is the number of elements in $\Omega$. Then, $\mu_\varepsilon \in \Delta^{|Z|}$ is a *stationary distribution* if $\mu_\varepsilon P(\varepsilon) = \mu_\varepsilon$. Kandori, Mailath and Rob (1993) note that the stationary distribution can be interpreted as the proportion of time that the process spends on each state. It should be said that, when the mutation rate $\varepsilon$ is positive, we have strictly positive transition probabilities ($p_{zz'}(\varepsilon) > 0$) between states $z$ and $z'$. This implies that the process has a unique stationary distribution[17]. Our goal is to characterize the *limit distribution* in the long-run which is defined by

$$\mu^* = lim_{\varepsilon \to 0}\ \mu_\varepsilon(.). \tag{7}$$

The *long-run equilibria* are the states which receive positive weight in the limit distribution. The set of *long-run equilibria* is defined by

$$LRE = \{z \in \Omega \mid \mu^*(z) > 0\}. \tag{8}$$

The limit or invariant distribution describes the long-run behavior of the learning process when the mutation rate is taken to be low. More precisely, the limit distribution realizes a selection between the long-run equilibria of the game. The key feature of this approach is the role played by mutations in being a source of perturbation of the selection dynamics. Note, however, that the lower the mutation rate, the longer the transition between the stationary states.

[17] See Kandori, Mailath and Rob (1993) or Young (1993).

# 3 Results of the model

The previous section focused on the description of the evolutionary model we use to capture the long-run behaviors in the monetary policy game. We now turn to the results of the model presenting both a formal proposition on the players behavior and its intuitive interpretation.

The outcome of the learning process largely depends on the assumptions regarding $1-\theta_i$ and $1-\theta_j$, the inertia level of both the government and the public. With no inertia (i.e., $\theta_i = \theta_j = 1$), each government player imitates in period $t$ the best government observed in $t-1$, and each public player reacts immediately to the choice of its government. In this case, the long-run equilibrium of the monetary policy game is clearly $z'' = (H,H)$. Consider the probabilities $p_{z'z''}$ and $p_{z''z'}$. When $\theta_i = \theta_j = 1$, the transition from $z'$ to $z''$ requires only one mutation of a government (i.e., $p_{z'z''} = \varepsilon$), whereas the transition from $z''$ to $z'$ requires at least two simultaneous mutations, one in each set of players (i.e., $p_{z''z'} = \varepsilon^2$). Then, we have $p_{z'z''} > p_{z''z'}$ as $\varepsilon \to 0$ and state $z'' = (H,H)$ is the long-run equilibrium.

This result is less obvious when $0 < \theta_i, \theta_j < 1$. This situation reduces the number of mutations required for the transition from $z''$ to $z'$ and the Pareto-optimal outcome one of the long-run equilibria in the monetary policy game. This result is stated by the following proposition.

**Proposition 2** *In the Monetary Policy Game, state $z'$ can be reached from $z''$ with only one mutation when inertia is included in the learning process, that is when $0 < \theta_i, \theta_j < 1$.*

**Proof.** Assume that the dynamics is in the state $z'' = (H,H)$. A single mutant government using $L$ can generate imitation in $I$ if this player, after the mutation, does not change his strategy (probability $1-\theta_i$) and at the same time the public he faces responds rapidly (probability $\theta_j$). Thus, the transition from $z'' = (H,H)$ to $z' = (L,L)$ can be carried out with only one mutation $L$ in the set of government players. ■

Proposition 2 underlines the importance of the learning opportunity in determining the outcome of the game. When inertia is included in the model, both states $z'$ and $z''$ are observed in the long-run, and not only the high inflation state.

It is worth noting that in some cases both the transition from $z''$ to $z'$ is made easier and in the same time the transition from $z'$ to $z''$ becomes harder. This situation favors the low inflation state in the long-run. It appears when the governments inertia is much higher than the publics reaction (i.e., when $1-\theta_i \gg 1-\theta_j$) and $T$ is sufficiently large. This is because in that case we can expect that $p_{z''z'}$ will be higher than $p_{z'z''}$. The two cases below underline this idea.

Fix $\theta_i < 1/n$ and $\theta_j = 1$. Consider first that the dynamics is in state $z'' = (H,H)$ and government $i$ plays $L$ by mutation at $t-1$. Following the learning rule, the public mutant $i$ faces plays $L$ as a best reply at $t$, and player $i$ becomes the government to imitate (since it realizes the highest payoff). Then, if $z^I \geq n/2$ before the random matching (what occurs for $T$ sufficiently large) then all the publics play $L$ at the beginning of the new $T$ periods ensuring that state $z' = (L,L)$ will be reached during this interval of time.

Consider now the other case where the dynamics is in state $z' = (L,L)$ and government $i$ plays $H$ by mutation at $t-1$. Then, the public confronted to player $i$ chooses $H$

at $t$. In the worst case, a government imitates player $i$ at beginning of $t$ (that is before the latter faces strategy $H$) and again is confronted to strategy $H$ at $t+1$. As we fix $\theta_i < 1/n$, this cycle ends because periods exist without learning regarding the governments[18]. Thus, we can argue that it is more likely to observe $z' = (L, L)$ in the long-run when both $1 - \theta_i \gg 1 - \theta_j$ and $T$ is sufficiently large.

As inertia given to the government and the public are the crucial parameters in our model (determining whether low inflation rates can be observed or not in the long-run), a macroeconomic interpretation is needed. When the private sector can adjust its strategy more frequently than the government can, the countries converge to the low inflation state is higher in probability. In the other case, too high inflation rates stay the best strategy to imitate since it offers the highest payoff (at least in the short and medium term). This result reflects that the resolution of governments to follow a strategy in the long-run (and their common belief that it is the best strategy) have played a crucial part in the disinflation process, even if this process can generate short-run costs[19].

Note also that only one mutation is required to observe the process reaching the low inflation state, from a state where all countries choose high inflation rates. This means it suffices that one country presents a high degree of inertia to show that low inflation is the best strategy. Then, it is imitated, but at this point the desinflationary policy in the other countries may be more or less costly according to the degree of public inertia. The stickier prices are, the costlier the desinflationary policy is, since a complete response to changes in monetary policy may require more than a single period.

The previous results have been stated under purely adaptive expectations. However, it has been showed by some authors that inflation expectations are merely captured by assuming the public is neither purely adaptive nor perfectly rational. The next section allows the evolutionary model to include partly rational expectations in order to discuss the relevance of previous results.

# 4 Partly rational expectations

So far we have considered that the public behavior is determined by adaptive expectations. In this case, the public is only backward looking and never expects changes in monetary policy. We now question the relevance of the model when a fraction of the public can make rational expectations. We think that this latter assumption is important regarding the recent literature on surveys of inflation expectations. This literature has come to mixed conclusions[20]. On the one hand, one result is that survey measures of inflation expectations are not purely adaptive - as we assumed in the previous section. On the other hand, survey expectations are not perfectly rational, since some players

[18]Notice that in this case the condition on $T$ is not necessary since the random matching leads back the dynamics to $z' = (L, L)$.

[19]This can be related to the lessons from experience of the European Monetary System. The strategic choice of the EMS countries for monetary and exchange rate stability has involved large differentials of unemployment rate and current account imbalances, illustrating the possible inconsistencies between short-term and long-term goals.

[20]See J. Roberts (1997).

do not make efficient use of available information[21].

In the spirit of Roberts (1997), we assume that expectations are halfway between rational and adaptive expectations. This is because the private sector is composed of well-informed agents (like investors in financial markets for instance) as well as poorly-informed ones. The market makers collect far more information than a normal citizen, and can react simultaneously in accordance with the government decisions.

Consider each private sector $j \in J$ as a large population of agents. Assume further that one fraction $x$ of $j$ follows the learning rule (5) whereas the other part $1-x$ has a perfect knowledge of the rationality and the payoff of the government $i \in I$. It follows that $1-x$ players know exactly the way the government chooses its strategy and the payoff realized. If we replace this assumption in our model then the previous result does not change.

Since the $1-x$ rational players know that the government imitates the best player $i \in I$ and that this process appears with probability $\theta_i$, then these players react more rapidly than the less rational ones. In case of disinflation, the cost can be significantly reduced as reaction to the low inflation strategy of the government is realized immediately by the perfectly rational agents.

It is important to note that in case of rational expectations the public knows that the government does not maximize an objective function, but rather follows an imitative process. As a consequence, the low inflation strategy turns out to be credible with rational expectations in our model (once this strategy has been experienced as the best), since the public knows the imitative process gives the low inflation strategy as the right choice. Moreover, the evolutionary analysis of the monetary policy game reveals that the inflationary bias in the Barro-Gordon model further relies on the perfect rationality given to the government rather than on the rational expectations assumption.

# 5 Conclusion

We want to conclude by assessing the main results of the model and relating them to the international monetary policy convergence within industrialized countries. In our evolutionary interpretation of the Barro-Gordon game, players simply take into account the decisions made in the past to determine their strategy. The learning process of governments is imitation of the best player, considering strategies used in the past by other governments; the private sector follows a best reply dynamics related to the strategy of the government. Levels of inertia respectively given to the government and the public are the crucial parameters in our model, determining whether low inflation rates can be reached or not in the long-run.

The model underlines the fact that the convergence of the countries to a low inflation state can be observed with a higher probability when the private sector adjusts its strategy more frequently than the government does, even if the public expectations are considered partially rational. This result reflects the importance of the government resolution to fight inflation and its ability to stick to that goal.

Relating to the historical process, the consensus on maintaining relatively low inflation arises from both previous success and conformity of public opinion. As for the history of the international monetary system, there have clearly been periods when

[21] See MH. Pesaran (1987).

large countries have exerted a stabilizing influence on the system. We can mention the case of Germany which began in the late 70s to make use of a tightened monetary policy. After the German commitment, an implicit contract emerged in the EMS where Germany (and its Bundesbank) followed macroeconomic policies that export price stability and anti-inflationary credibility to the other countries. The latter have to support both their exchange rate obligations and their inflation objectives. This results from price stability viewed by all decision-makers as the appropriate primary goal of monetary policy, coming from the inability to influence real outcomes in the long run. This common belief also concerns public opinion because the primacy of price stability acknowledges the primacy of the public's assessment of when inflation is hurting them.

The main contribution of the present paper is to demonstrate the monetary policy convergence process to a low inflation level when the government choice follows an imitation process. This evolutionary analysis lies beyond the strict game-theoretic methods which focus on the policy coordination and treat the participating governments as antagonists engaged in policy barter. Moreover, this evolutionary approach does not need to consider in the monetary game the presence of explicit institutional arrangements concerning monetary institutions for reaching price stability. Rather, the evolutionary forces show that the resolution and the belief of governments in low inflation benefits are the main components for successful policies. However, it could be interesting to use evolutionary approach to explain why central bank independence has increased so much over the last decade (a process which culminated with the European Central Bank creation). Central bank independence could be then interpreted as an imitation of the best player, namely Germany.

It should also be stated that we don't want to initiate a critical analysis of the Barro-Gordon model and the associated central bank independence issue. Above all we want to show the low inflation process as a social choice, involving both governments and private sectors, which depend on the belief of price stability as a primary goal. In a manner suggested by Taylor (1982), McCallum (1995) or Ireland (1999), we demonstrate that the monetary authority acts in order to build credibility for a disinflationary policy by simply adopting and following that policy for a sufficient length of time, and it will stick to that policy even if it imposes short-run costs on the economy.

# References

al Nowaihi, A. & Levine, P. (1994), 'Can reputation resolve the monetary policy credibility problem', *Journal of Monetary Economics* **33**, 355–380.

Backus, D. & Driffill, J. (1985), 'Rational expectations and policy credibility following a change in regime', *Review of Economic Studies* **LII**, 211–221.

Barro, R. & Gordon, D. (1983), 'Rules, discretion, and reputation in a model of monetary policy', *Journal of Monetary Economics* **12**, 101–121.

Blinder, A. (1999), Central bank credibility: Why do we care? how do we build it?, Working paper, NBER.

Cukierman, A. (1987), *Central bank strategy, credibility and independence*, Cambridge MA: MIT Press.

Hayo, B. & Hefeker, C. (2001), ‘Central bank independence: A survey and evaluation’, *European Journal of Political Economy* . Forthcoming.

Hillman, A. (1987), *Political Culture and the Political Economy of Central Bank Independence,*, in Blejer and Streb (eds), Major Issues in Central Banking, Monetary Policy, and Implications for Transition Economies, Amsterdam, Kluwer.

Ireland, P. (1999), Expectations, credibility, and time-consistent monetary policy, Working paper, Boston college and NBER.

Kandori, M., Mailath, J. & Rob, R. (1993), ‘Learning, mutation, and long-run equilibria in games’, *Econometrica* **61**, 29–56.

Kreps, D. & Wilson, R. (1982), ‘Reputation and imperfect information’, *Journal of economic Theory* **27**, 253–279.

Kydland, F. E. & Prescott, E. C. (1977), ‘Rules rather than discretion: The inconsistency of optimal plans’, *Journal of Political Economy* **85**, 473–91.

Lohmann, S. (1992), ‘Optimal commitment in monetary policy: credibility versus flexibility’, *American Economic Review* **82**, 273–86.

McCallum, B. (1995), ‘Two fallacies concerning central-bank independence’, *American Economic Review* **82**, 273–286.

Persson, T. & Tabellini, G. (1993), ‘Designing institutions for monetary stability’, *Carnegie-Rochester Conference Series on Public Policy* **39**, 53–84.

Pesaran, M. (1987), *The Limits to Rational Expectations*, Basil Blackwell. Oxford.

Roberts, J. (1997), ‘Is inflation sticky?’, *Journal of Monetary Economics* **39**, 173–196.

Rogoff, K. (1985), ‘The optimal degree of commitment to an intermediate monetary target’, *Quartely Journal Of Economics* **100**, 1169–1190.

Selten, R. (1975), ‘Reexamination of the perfectness concept for equilibrium points in extensive games’, *International journal of game theory* **4**, 25–55.

Selten, R. (1978), ‘The chain-store paradox’, *theory and decision* **9**, 127–159.

Taylor, B. (1982), ‘Establishing credibility: a rational expectations viewpoint’, *American Economic Review* **72**, 81–85.

Vega-Redondo, F. (1996), *Evolution, games, and economic behaviour*, Oxford University Press.

Walsh, C. (1995), ‘Optimal contracts for central bankers’, *American Economic Review* **85**, 150–67.

Weibull, J. (1987), *Evolutionary game theory*, MIT Press, Cambridge, MA.

Young, P. (1993), ‘Evolution of conventions’, *Econometrica* **61**, 57–84.