Economic Growth Exercises

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1 Basic concepts and tools

Basic concepts

Question 1. *Give the definition of the following concepts : GDP, per capita GDP and GDP/worker.*

Question 2. *Determine γ_Y if $Y_t = (aX_t/Z_t)^\alpha$. Note : $a > 0$ and $\alpha \in [0,1]$ are both constant in time.*

Question 3. *Give the evolution in time of the variable N_t if this evolution happens at a constant instantaneous rate m. Plot this variable against time, if its initial value is N_0. Same question for $logN_t$.*

Question 4. *Country A's current GDP/worker is 4000€ and its growth rate is 5%. Country B, has a GDP/worker of 3000€ and a growth rate of 8%. What can you say about the evolution in time of the distance between these countries in terms of per capita income ? Plot $logY_t^A$ and $logY_t^B$ against time.*

Question 5. *Explain the following concepts and the relationships that could exist between them : steady state, balanced growth path and stationary state.*

2 Keynesian and post-Keynesian analyses

Question 6. *Consider a country producing only one good, whose level is, at each instant Y_t, combining capital and labour as complementary factors. For producing one unit of output, at least 3 units of capital must be combined with at least two units of labour. The population grows at the instantaneous rate of 5%, and the participation rate is 100%. The households save 9% of their income. There is no depreciation of capital, nor technical progress in this economy. Initially (at $t = 0$), the economy disposes of $K_0 = 150$ units of capital and $L_0 = 100$ units of labour.*

1. *Determine the initial level of production Y_0. Same question if $L_0 = 200$. Idem if $K_0 = 250$. What would be the level of unemployment in all these three cases ? What can you say about unemployment at $t > 0$ given the initial state of the economy indicated in the question, if the capital remains at K_0 ?*
2. *Determine the labour demand in this economy, and the condition of equilibrium on the labor market. What would be the growth rate of the GDP γ_Y^L that would be compatible with this equilibrium ? Draw the plot of $logY_t$ against time in this case, starting from Y_0 determined above (with $K_0 = 150$ and $L_0 = 100$) . What happens if the current growth rate is $\gamma_Y < \gamma_Y^L$? Can we have $\gamma_Y > \gamma_Y^L$?*
3. *Taking into account the fact that firms invest in capital in order to follow the expected variations of the GDP (corresponding to the accelerator mechanism), and they have perfect foresights, determine the investment behavior of the economy.*
4. *The economy is closed and all income saved by the households is the only resource for financing investment in this economy. Under which condition, the investment desired by the firms could be financed in this economy ? What would be the growth rate of the GDF $\left(\gamma_Y^K\right)$ that would be compatible with this condition. Draw the plot of $logY_t$ against time in this case, starting from Y_0 determined above.*
5. *Comparing the plots of questions questions 2 and 4, determine the evolution of unemployment in time in this economy. What should be the saving rate in order to assure the balanced growth path of the economy ?*

Question 7. *Consider a country producing only one good, whose level is at each instant Y_t, combining capital and labour as complementary factors. For producing one unit of output, at least 3 units of capital must be combined with at least two units of labour. The population grows at the instantaneous rate of 5%, and the participation rate is 100%. There is no depreciation of capital, nor technical progress in this economy. Initially (at $t = 0$), the economy disposes of $K_0 = 150$ units of capital and $L_0 = 100$ units of labour. All income in this economy is distributed as wages (W) or profits (P). This economy is composed of two classes : the workers who receive as income the wages, and the capitalists who receive the profits. Workers save a fraction $s_w = 3\%$ of their income and the capitalists, a fraction $s_c = 10\%$.*

1. *Determine the total amount of savings (S_t) in this economy and each moment, and the corresponding aggregate saving rate (S_t/Yt).*

2. *If the firms desire to invest $I_0 = 1$, what should the share of the income Y_0 be distributed as profits in order to ensure the initial equilibrium of the good market ? Same question with $I_0 = 10$, and $I_0 = 2.5$.*
3. *Determine the investment rate (I/Y) that this economy should adopt on the BGP in order to ensure the full employment of the active population.*
4. *Determine the share of the income that should be distributed as profits to ensure this full-employment investment rate. Same question if $s_c = 15\%$.*

3 Solow

Basic Solow model

Question 8. *Consider a closed economy producing a single good, using the technology represented by the following production function :*

$$Y = AK^{\alpha}L^{1-\alpha}$$

The households save 10% of their income, and their population grows at an instantaneous rate of 5%. The capital stocks depreciates at an instantaneous rate of 2%. Econometric studies indicate that $\alpha = 1/3$ and $A = 1$.

1. *This country currently possesses a capital stock of $K_0 = 1000$, and a population of $L_0 = 1000$. Analyze the evolution in the short term of this economy (Trick : the growth rate of the capital/worker could guide you).*
2. *Same question if $K_0 = 4500, L_0 = 1500$.*
3. *And if $K_0 = 2500, L_0 = 100$.*
4. *What could you conclude from these studies on the relationship between the initial income of the economy and the corresponding* per capita *growth rate ?*
5. *What would the state of this economy become in the long term ? Determine its capital/worker, GDP/worker and consumption/worker in that state. Indicate the growth rate of these variables, and also of the GDP and the consumption.*
6. *We observe that the households have just changed their saving behavior, and they save now 20% of their income. What would be the evolution of this economy in the short term, and its state in the log term ? Analyze the evolution of the GDP/worker and the consumption/worker in the short term and the long term.*
7. *What could you conclude from the comparison of your answers to the last two question on the relationship between the saving behavior of this economy, and its income level ?*
8. *The economy starts to ameliorate its technology, and we can observe this phenomenon because A increases now at an instantaneous rate of 2%. Answer the question 6, taking into account this new development.*

Basic growth accounting

Question 9. *In a country under study, you observe that the production can be analyzed using the following aggregate production function:*

$$Y = F(K, L) = AK^{1/3}L^{2/3} \tag{1}$$

You also have statistics on this economy covering three or three decades, and giving the average growth rates of some variables:

Period	Y	K	L	y
$60-70$	*5*	2	3	
$70-80$	*3*	*1*	*2.5*	
$80-90$	*4.5*	*2*	*2*	

Average growth rates (%)

1. *Show that the growth rate of the GDP in this economy can be decomposed using the growth rates of the variables appearing in the right hand of the equation* (1).
2. *Complete the last column of the table, for the GDP/worker.*
3. *Analyze the factors that could explain the growth rates of the GDP/worker* (γ_Y) *in these three decades, and explain why this growth rate has been different in them.*
4. *Can the variations of K and L completely explain the variations of Y? If not, what is missing ? If necessary, complete the table by including the missing factor and discuss your results.*

Solow meets Keynes

Question 10. ***Equilibrium and disequilibrium in the Solow Model***

Consider a closed economy producing a single good, using the technology represented by the following production function:

$$Y = AK^{\alpha}L^{1-\alpha}$$

The households save a fraction s of their income, and their population grows at an instantaneous rate of n. The capital stocks depreciates at an instantaneous rate of δ. Econometric studies indicate that $\alpha = 1/3$ and $A = 1$.

1. *Determine how the capital stock of the economy varies in each instant* $\left(\dot{K}\right)$.
2. ***The investment desired by the firms*** *depends on their expectations on the economy, and corresponds to the following behavioral equation:*

$$I_t^d = \beta K_t. \tag{2}$$

What would be the dynamics of the capital, if we assume that the economy can finance this desired investment, and if we have $\beta < \delta$? Same question if $\beta \geq \delta$. Deduce from this analysis the equation of per capita investment $\left(i^d \equiv I^d/L\right)$.

3. *Given that this economy is a closed one, how should it finance this desired investment? Is it assured to be able to finance it ? If not, what would be consequences on the equilibrium of the good market?*
4. *Determine the level of capital/worker that would be observed at the equilibrium of the good market (k^e), and, the* ***actual level of investment*** *in this country, as a function of k. Comparing the per capita investment and per capita saving on a diagram could help you.*
5. *Determine the fundamental dynamic equation of this economy, taking into account the actual level of investment that you have determined in the previous question.*
6. *Build the Solow diagram corresponding to this economy, and analyze the dynamics of the economy, and its BGP in the following three cases:*
   a) $\beta > n + \delta$
   b) $\beta = n + \delta$
   c) $\beta < n + \delta$
7. *Compare your results with the ones of the Solow model.*
8. *Same question for the Keynesian model of Harrod.*

Solow meets Leontieff

Question 11. *Consider the Solow model, but take into account the fact that the production technology is of Leontieff type:*

$$Y = min\{aK, bL\}$$

the saving rate of the economy is s and the population grows at rate n. There is no depreciation of capital.

1. *Build the intensive form of this production function, $y = f(k)$. Represent it in the frame (k, y).*
2. *Analyze the dynamics of the economy using the Solow diagram. Consider three cases: a) $s \cdot a < n$; b) $s \cdot a > n$; c) $s \cdot a = n$*
3. *Compare the behavior of this economy with the one you would observe in a simple Keynesian framework* à la *Harrod.*

Choosing technologies...

Question 12. *Consider the Solow economy where the producers can use a tradition technology,*

$$Y_O = AK^{\alpha}L^{1-\alpha}$$

but also a more modern technology

$$Y_M = \Omega K^{\alpha}L^{1-\alpha}, \Omega > A$$

In order to exploit this better technology, the country must pay an exploitation cost (eg. corresponding to a necessary public infrastructure) in every period. This cost is proportional to the labor force, and is given by βL. The government collects a tax β from each worker in each period, and may use it to finance this exploitation cost.

1. *Write the intensive form of the production functions corresponding to each technology, net of the exploitation cost.*
2. *Which technology the producers would use if the government choses to pay the installation cost using the collected taxes?*
3. *Should it pay the setup cost for any level of k? If not, determine when it would be reasonable for it to pay it.*
4. *Draw these production functions in the frame (k, y), taking into account your previous answers.*
5. *Analyze graphically the evolution of average productivity using the slope of the cord that goes from the origin to the effective production function, taking into account the potential switching to the new technology.*
6. *Determine the growth rate of k given these results, and determine its features by confronting the saving on average productivity and the social depreciation in a diagram.*
7. *Determine the steady states of this economy. Study their stability and discuss the possibilities of growth in this country.*

I can get no...

Question 13. *Consider the Solow economy where the technology is given by the following production technology*

$$Y = AK$$

where K represents now a composite capital, including also the human capital. Hence the per capita output is given by $y = Ak$. A represents the technological level of the economy, and it is constant.

1. *Is this a neoclassical production function? Which properties does it verify? Which ones does it not verify, if any? Discuss the properties of this production technology.*
2. *Determine the BGP of this economy, and discuss its properties.*

4 Education, human capital and growth

Simple application of MRW

Question 14. *A country where:*

$$Y = K^{\alpha} (AH)^{1-\alpha}$$

$\alpha \approx 1/3$, $n = 5\%, g \equiv \dot{A}/A = 2\%$, $u = 20\%$, $\psi = 10\%, s = 20\%, \delta = 0$.

1. *Fundamental dynamic equation of the economy?*
2. *y_t^*?*

Empirical application of simple MRW

Question 15. *We observe the following values:*

Country	y_{90}	s_K (%)	u (%)	n (%)	$\widehat{A}_{90}$
USA	2000	21	11.8	0.9	1.00
Canada	1620	25.3	10.4	1	
Brazil	500	16.9	3.7	2.1	
China	1200	22.2	7.6	1.4	
Kenya	100	12.6	4.5	3.7	

We assume that other characteristics of the economies are common: $\alpha = 1/3, \psi = 0.10$ *and* $g + \delta = 7\%$. *Compare the relative GDP/worker levels predicted by MRW and the observed values in two cases:*

(a) *All countries have the same TFP.*

(b) *Using the estimation of* $\widehat{A}_{90}$ *for each country.*

(c) *What could you .conclude from the comparison of the previous two answers?*

5 Ramsey model

Question 16. *Consider an economy à la Ramsey where we have:*

$$u(c) = c^{\beta}, f(k) = k^{\alpha}$$

$\alpha = 1/2, \beta = 1/3, n = 0.05, \theta = 0.2.$

1. *Write the problem of the planner maximizing the utility flow of the households.*
2. *Build the current value Hamiltonian corresponding to this problem.*
3. *Determine the sufficient conditions for the resolution of this problem.*
4. *Determine the level of per capita consumption* $c*$ *and per capita capital* k^* *corresponding to the optimal BGP of this economy.*
5. *Draw the phase diagram around this BGP, and study the dynamics of the economy and the stability of the BGP.*
6. *Determine the saving rate of the economy at the BGP.*