The footprint of evolutionary processes of learning and selection upon the statistical properties of industrial dynamics

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Abstract

Evolutionary theories of economic change have identified as the two main drivers of the dynamics of industries the mechanisms of *market selection* and of *idiosyncratic learning* by individual firms. In this perspective, the interplay between these two engines shapes the dynamics of entry-exit and the variations of market shares and, collectively, In the following contribution we shall present a simple agent based model, formalizing barebone mechanisms of learning and selection, able to robustly yield an ensemble of empirical stylised facts, including ample heterogeneity in productivity distributions, persistent market turbulence and fat tailed distributions of growth rates.

Keywords

Firms Growth Rate, Productivity, Fat Tail Distributions, Learning Processes, Market Selection Mechanism.

JEL Classification

C63-L11-L6

1 Introduction

Evolutionary theories of economic change have identified as the two main drivers of the dynamics of industries the mechanisms of *market selection* and of *idiosyncratic learning* by individual firms. In this perspective, the interplay between these two engines shapes the dynamics of entry-exit and the variations of market shares and, collectively, productivity and size distributions and the patterns of growth of productivities and sizes themselves. Learning (what in the empirical literature is sometimes broadly called the *within effect*) stands for a various processes of idiosyncratic innovation, imitation, changes in technique

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of production. Selection (what in the empirical literature is called the *between effect*) is the outcome of processes of market interaction where more competitive firms gain market shares at the expense of less competitive ones (or not), some firms die and others enter. The ensuing industrial dynamics presents some remarkable and quite robust statistical properties – "stylized facts" – which tend to hold across industries and countries, levels of aggregation and time periods (for a critical survey, see Dosi et al., 1995).

In particular, the stylized facts include:

- persistent heterogeneity in productivity and all other performance variables;
- persistent market turbulence due to change in market shares and entry-exit phenomena;
- skewed size distributions;
- fat tail distribution of growth rates;

Different theoretical perspectives address the interpretation of one or more of such empirical regularities. One stream of analysis, which could go under the heading of *equilibrium evolution*, try to interpret size dynamics in term of *passive* (Jovanovic (1982)) or *active* learning (Ericson and Pakes (1995)). Another stream – from the pioneering work by Ijiri and Simon (1977) all the way to Bottazzi and Secchi (2006a) – studies the result of both mechanisms in terms of the ensuing exploitation of new business opportunities, captured by (?) together of learning and selection.

A second stream, including several contributions by Metcalfe (see among others Metcalfe (1998)), focuses on the processes of competition/selection often represented by means of a replicator dynamics where shares vary as a function of the relative competitiveness or "fitness".

Finally, many evolutionary models unpack the two drivers of evolution distinguishing between some idiosyncratic processes of change in the techniques of production, on the one hand, and the dynamic of differential growth driven by differential profitabilities and the ensuing rates of investment on the other (such as in Nelson and Winter (1982)). Yet, in other evolutionary models selection is represented by means of an explicit replicator dynamics, (such Silverberg et al. (1988) and).

In the following contribution we shall present a simple agent based model, formalizing barebone mechanisms of learning and selection, able to robustly yield an ensemble of empirical stylised facts, including ample heterogeneity in productivity distributions, persistent market turbulence and fat tailed distributions of growth rates.

In particular in section 2 we will briefly summarize the empirical stylized facts, in section 3 we will discuss the main theoretical models aimed at explaining some of them, in section 4 we will present our model and, in section 5 we analyse the simulation results.

2 Empirical stylised facts: productivity, size and growth

During the last few decades, allowed by the availability of longitudinal micro data, an increasing number of studies have identified a rich ensembles of stylised facts related to *productivity* and *size* distributions, and firm *growth rates*. Let us consider some of them, germane to the model which follows.

2.1 Productivity distribution and growth

As extensively discussed in Doms and Bartelsman (2000), Syverson (2011), Dosi (2007) and Foster et al. (2008) among many others, *productivity dispersion*, at all levels of disaggregation, is a striking and very robust phenomenon. Moreover such heterogeneity across firms is *persistent over time*, (cf. Bartelsman and Dhrymes (1998), Dosi and Grazzi (2006) and

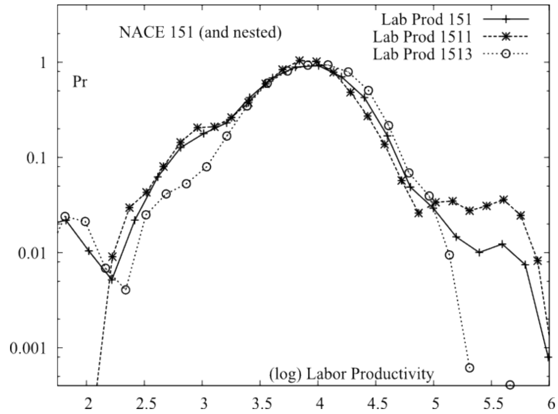


Figure 1: Empirical distribution of labour productivity. Source: Dosi et al. (2013).

SECTOR	ISIC Code	Labor Prod. AR(1)	Labor Prod. Std.Dev.	Π_l Growth rates AR(1)	Π_l Growth rates Std.Dev
Production & processing of meat	151	1.0021	0.0016	-0.3446	0.0915
Knitted & crocheted articles	177	1.0056	0.0023	-0.2877	0.1005
Wearing apparel & acc.	182	1.0035	0.0012	-0.3090	0.0871
Footware	193	1.0029	0.0019	-0.3903	0.0793
Articles of paper and paperboard	212	1.0053	0.0008	-0.3027	0.0603
Printing and services related to printing	222	0.9962	0.0011	-0.4753	0.1103
Plastic products	252	1.0030	0.0010	-0.3150	0.0557
Articles of concrete, plaster & cement	266	0.9985	0.0016	-0.4572	0.0979
Metal products	281	1.0034	0.0012	-0.4125	0.0715
Treatment, coating of metal & mech. engin.	285	1.0051	0.0013	-0.1846	0.0679
Special purpose machinery	295	1.0011	0.0011	-0.3040	0.0495
Furniture	361	0.9994	0.0001	-0.4472	0.0808

Figure 2: AR(1) coefficients for Labour Productivity in levels and first differences, Italy, Istat Micro.1 Dataset. Dosi and Grazzi (2006).

Bottazzi et al. (2008)) with autocorrelation coefficients in the range $0.8 - 1$. An illustration is provided in figure 1 for one 2-digit Italian sector and two 3-digit thereof: notice the wide support of the distribution that goes from 1 to 5 in log terms. The distribution and its support are quite stable over time and so is the "pecking order" across firms as suggested by the high autocorrelation coefficients: see Table 2. These findings, robust to the use of parametric and non-parametric tools, empirically discard any idea of firms' revealed production process as the outcome of an exercise of optimization over a commonly shared production possibility set, which, under common relative prices, ought to yield quite similar input/output combinations. Rather *asymmetries* are impressive, which tells of a history of both firm-specific learning patterns and of co-existence in the market of low and high productivity firms with no trace of convergence (see Dosi et al. (2012)). Even more so, asymmetries are pronounced in emerging economies (with some reduction along the process of development): see on China, Yu et al. (2014). In that, the survival of dramatically less efficient firms hint at the structurally imperfect mechanism of market selection, which demand a theoretical interpretation.
A less explored phenomenon related to the dynamic of productivity is the *double exponential nature* of its growth rate distributions. Extensive evidence is provided in Bottazzi et al. (2005) and Dosi et al. (2012): see figure 3 for an illustration. The double exponential nature of growth rate in productivity does not only reveal an underlining multiplicative process that determines efficiency changes, but also hint processes of idiosyncratic learning characterized by discrete, relatively frequent "big" events (see also below on growth rates in size).

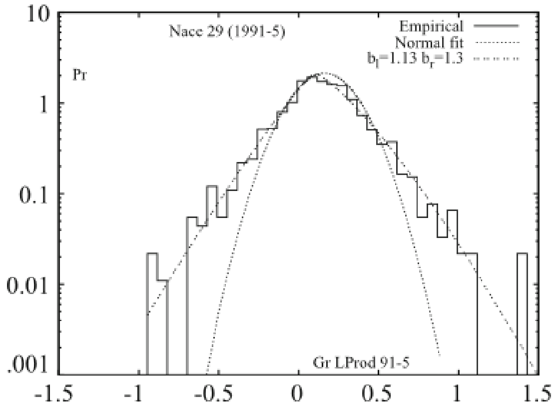


Figure 3: Tent shaped productivity growth rate. Italy, Istat Micro.3 Dataset. Source: Dosi et al. (2012).

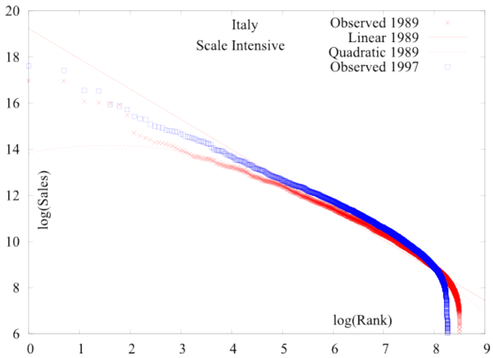


Figure 4: Skewness in the size distribution. Source: Dosi et al. (2008).

2.2 Size distribution

Firm size distributions are skewed. And this is an extremely robust property which, again, holds across sectors, countries and levels of aggregation. But, how skewed is skewed?
The *power-law nature* (Pareto or Zipf law according to the slope of the straight line, in a log-log plot) [1] of the firm size distribution, has been investigated by many authors[2] since the pioneering work by Simon and Bonini (1958). This is not the place to discuss the possible generating mechanism of such distribution (an insightful discussion is in Brock (1999)). Here, just notice that, being the Pareto a scale free distribution, in principle it should be scale invariant, or equivalently, it should be detectable irrespectively of the considered level of aggregation. However Bottazzi et al. (2007), Dosi et al. (2008) find that the size distributions fairly differ across sectors in terms of shape fatness of the tails and even modality. Plausibly, technological factors, the different degree of cumulativeness in the process of innovation, the predominance of process vs product innovation might strongly affect sector specific size distributions (Marsili, 2005). It might well be that the empirical findings on the Zipf (Pareto) law distribution is a mere effect of the aggregation as already discussed in Dosi et al. (1995). What should be retained is the skewness of the distribution (see figure 4).

[1] See Newman, 2005 for a succinct overview.
[2] See Stanley et al., 1995 and Axtell, 2001 for US manufacturing data.

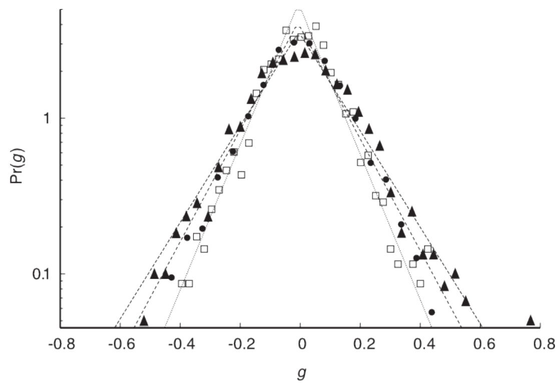


Figure 5: Tent shaped size growth rate, Italian, Istat Micro 1 Dataset. Source: Bottazzi and Secchi (2006a).

2.3 Market turbulence

Underneath the foregoing invariances, however, there is a remarkable turbulence involving changes in market shares, entry and exit (cf. the discussion in Baldwin and Rafiquzzaman (1995) and Doms and Bartelsman (2000)). A good deal of such turbulence is due to "churning" of going (???) firms, with $20\% - 40\%$ of enters die in the first two years and only $40\% - 50\%$ survive beyond the seventh year in a given cohort.

2.4 Fat tailed distributions of growth rates

A huge empirical literature testifies the emergence of *Laplace distribution* in growth rates. A typical empirical finding is illustrated in 5. This applies across different levels of sectoral disaggregation, across countries, over different historical periods for which there are available data and it is robust to different measures of growth, e.g. in terms of sales, value added or employment, (for more details see Bottazzi et al. (2002), Bottazzi and Secchi (2006a), Bottazzi et al. (2008) and Dosi (2007)).
Firms grow and decline by relatively lumpy jumps which cannot be accounted by the cumulation of small, - "atom-less"-, independent shocks. Rather "big" episodes of expansion and contraction are relatively frequent. More technically, this is revealed by fat tail distributions (in log terms) of growth rates. What determines such property?
In general, such fat tail distributions are powerful evidence of some underlying correlation mechanism. Intuitively, new plants arrive or disappear in their entirety, and, somewhat similarly, novel technological and competitive opportunities tend to arrive in "packages" of different "sizes" (i.e. economic importance). This is what Bottazzi (2014) calls the *bosonic nature* of firm growth, in analogy with the correlating property of a family of elementary particles – indeed the bosons –.

2.5 Scaling growth-size relation

The multiplicative process of firm growth is (roughly) uncorrelated with growth, at least for not too small firms. This is what under the heading of the *Gibrat law*.
However, the *variance* of growth rates falls with size. How the level and the size growth rate are related one to each other? Since Stanley et al. (1996) an extensive literature of empirical papers found a negative relation between the variance of growth rates and size (see Sutton (2002), Lee et al. (1998), Bottazzi and Secchi (2006b)). An illustration of the phenomenon is provided in figure 6. The underlining idea is that firms can be described as a collection of independent units, each of them characterised by a growth process. The higher the firm size,

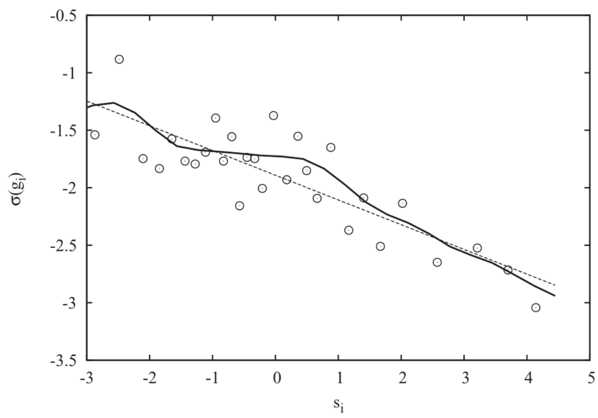


Figure 6: Variance-size relation. Elaboration on PHID, Pharmaceutical Industry Dataset. Source:Bottazzi and Secchi (2006b).

the higher the number of its components ("lines of business"); hence under an assumption of independent growth processes for each of the unit, the variance of the growth rate decreases proportionally to the inverse square root of size.

To summarize, the challenge for the theory is to account for: [i] persistent heterogeneity in efficiencies, [ii] skewness in size distribution, [iii] market turbulence, [iv] fat tail distributions of growth rates, [v] negative variance-size relations. How do incumbent theories face in this respect?

3 Theoretical interpretation

Let us start from that family of models which we call in Dosi et al. (1995) of "equilibrium evolution" including Jovanovic (1982), Ericson and Pakes (1995), Hopenhayn (1992a), Hopenhayn (1992b) and Pakes and Ericson (1998) there is the explicit attempt to link heterogeneity among firms, their growth, and possibly survival. The evolutionary aspects lies in the idiosyncratic productivity process that leads more productive firms to expand their own capacity, or equivalently their market shares, and less productive firms to shrink their weight in the market up their death. Implicitly, there is some process of selection that reward the more efficient and penalize the less efficient at work. However, competitive selection never appears because rational agents "learn" their equilibrium size. These models, at a first glance distinguishable in passive (as Jovanovic (1982)) and active (as Ericson and Pakes (1995)) learning models, address issue as growth/death rates conditional on age, the dependence or not of the current size on the initial one, the entry-exit rate in equilibrium. The "rational" attributes stems from the fact that all of them are characterized by profit-seeking maximizing agents over an infinite time horizon that can in each time step "decide" their equilibrium size and whether to stay in the market or not according to their technological rational expectations. Thus, in fact, selection is never at work because it is, so to speak, "anticipated in the heads of the agents" and, as a consequence every observed variable is an equilibrium one: hard to believe indeed! Moreover, this class of models seems unable to offer any guidance on the interpretation of productivity and size distributions (except the rationalization of whatever observation as an equilibrium one) and of the ubiquitous fat-tailedness of growth rates.
The abandonment of strong rationality and equilibrium assumptions bears abundant fruits. Simple characterization of firm growth as a multiplicative process, such as in the seminal Ijiri and Simon (1977), already yield insights into the determination of observed size distributions. Ijiri and Simon (1967) the authors decompose the total growth rate as the sum

of an idiosyncratic component and an industry time-variant component. The growth rate at a sectoral level is assumed to be constant, as the initial size of the firm. Finally the idiosyncratic shock is modelled as an $AR(1)$ process capturing [i] the independence of growth rate from size (Gibrat Law), [ii] one-period autocorrelation of growth rates (or single period Markow process), [iii] a mean reversion behaviour. The growth rate is described as the sum of independent micro-shocks. The size path turns out to be a random walk model with a drift yielding log-normal distributions.

However, the interactiveness intrinsic to the dynamics of firm growth is still missing. And this is what different families evolutionary and "Simonesque" models are meant to capture. In such a perspective, the micro-patterns of the industrial dynamic are the outcome of twin processes of *learning* and *selection* among *boundedly-rational, interacting agents*. The aggregate regularities emerge as the result of the continuum coupling of *change* and *coordination*. Models like the ones proposed in Nelson and Winter (1982), Silverberg and Lehnert (1993), Dosi et al. (1995), Dosi et al. (2000), Bottazzi et al. (2001) and Winter et al. (2003) are some different and complementary examples of the evolutionary-modelling approach. Evolutionary models that address the pattern of industry evolution with particular reference to the learning and selection process, can be subdivided into three different categorizations: [i] the group of models that mainly focus on the selection process (see Metcalfe (1998) for an extensive discussion), [ii] the group of models that mainly investigate the pattern of firm growth as a process of cumulation of learning opportunities (from Ijiri and Simon (1977) to Bottazzi and Secchi (2006a)) folding together the two different processes; [iii] the group of models that unpack and treat separately the two processes (Silverberg et al. (1988) and Dosi et al. (1995)).

Of particular interest for our purposes here, are those "Simonesque" models which parsimoniously but powerfully account for correlating mechanisms both on the learning and the competition, such as, Bottazzi and Secchi (2006a), nesting such correlations into firm specific increasing returns in (correlated) business opportunities. Building upon the "island" model by Ijiri and Simon (1977), they introduce the hypothesis of exploitation of business opportunity via a Polya Urn scheme, wherein in each period "success breeds success". It is a two step model, where in the first step an assignment procedure of the *fixed number* of business opportunities is realised. In the second step, these business opportunities act as source of growth. The dynamic of firms growth rate is still al a *Gibrat-type* but with the strong difference that the number of opportunities M are not assigned with a constant probability $1/N$, but proportionally to the number of opportunities that in *each period* the firm already has access to, which is correlated to the opportunities of all other firms. At each time step a micro-shocks of type $i \in 1, ..., N$ is extracted from an urn. Once it is extracted the ball is replaced and, additionally, a new ball of the same colour is introduced. This implies that, once one type i has been extracted, the probability of being re-extracted increases. This procedure is repeated M times, the number of the total business opportunities. Indeed this cumulative process is at the core of the emergence of fat tails distributions. Bottazzi and Secchi (2006a) demonstrate that when N and the ratio M/N increase, the limit distribution of this scheme is Laplace distributed, and this occur independently from the distribution function of the shocks. This explanation, which we share indeed, of the tent shape relies on the idea that a big chunk of microshocks M are concentrated in few firms N. In this representation however a significant drawback is that the assignment procedure occurs once a year: dynamic increasing returns displace in *space* (that is, cross-sectionally), as a cumulation of many shocks in few firms, but never in *time*. It turns to be rather difficult imagine that firms update their expertise every year, when the urn is open.

Our conjecture, however, is that synchronic cumulative processes are only one of the drivers of the apparent correlations underlying the "tents" in growth rates. Indeed, we suggest that a rather large ensemble of evolutionary processes, characterized by different forms of *idiosyncratic* (i.e. firm- specific) *learning* and *competitive interactions* yields the observed distributions of growth. This is the conjecture we are going to explore in this work. And we shall show an explicit account of learning and selection dynamics is able to yield *also* to robust accounts of the stylised facts flagged earlier, such as productivity and size distributions, autocorrelation in asymmetries in efficiency and micro turbulence.

4 The model

The model is an evolutionary agent-based model microfounded upon simple behavioural-heuristics. The absence of any rational technological expectation is intentionally pursued. The most part of human decisions, and among them economic decisions as putted by Gigerenzer and Selten (2002) are made under low degree of information, time pressure, uncertainty and low computational effort. For these reasons the low of motion that govern our economy are not derived from the profit seeking maximizing agents. They are conversely, empirically grounded.
The three processes that takes place in the market are learning, selection and entry. The model, as we will show, is able to provide a rich ensemble of stylised facts of the pattern of industrial evolution.

4.1 Idiosyncratic learning process

We build upon a simplified version of Dosi et al. (1995) whereby learning is represented by some multiplicative stochastic process upon firms productivity or more generically "level of competitiveness" a_i of the form:

$$a_i(t) = a_i(t-1)(1 + \theta_i(t)) \tag{1}$$

where the $\theta_i(t)$ are the realization of the firm-specific process. Formally θ_i are realization of a sequence of random variables $\{\Theta_i\}_{i=1}^{N}$ where N are the fixed number of firms. This equation aims to capture the dynamic of capabilities formation within each firm. According to the emerging-capabilities literature (see Teece et al. (1994)) firms stuck in their attitude to do innovation, searching, problem solving and so on. The internal capability structure reflects into the external productivity dynamics, embedded in the ability to be more competitive (lower prices via process innovation) or to introduce new products. Different capability structures are actually considered as the main source of heterogeneity among firms. The choice of a multiplicative process to model the dynamic of productivity is basically meant to grasp its persistent heterogeneous nature across firms and it turns to be a *Gibrat-type* dynamic not as usual in size but in the level of competitiveness.

We experiment with different learning processes:

- $\theta_i(t)$ is drawn from a set of possible alternative distributions namely Normal, Lognormal, Poisson, Laplace and Beta, namely a *Baseline Regime*;

- $\theta_i(t) = 0$ under *Schumpeter Mark I*;

- $\theta_i(t) = \pi_i(t)\left(\dfrac{a_i(t-1)}{\sum_i a_i(t-1)s_i(t-1)}\right)^{\gamma}$ under *Schumpeter Mark II*, where $\pi_i(t)$ is the same draw as under the *Baseline Regime*

At one extreme, in the first case, incumbents do not learn after birth. Advances are only carried by new entrants. At the opposite extreme, in the third case, incumbents do not only learn, but do it in a cumulative way so that a "draw" by any firm is scaled by its extant relative competitiveness. This captures what Paul David, quoting Robert Merton, calls the

"Matthew effect":
"For unto every one that hath shall be given, and he shall have abundance: but from him that hath not shall be taken even that which he hath." (Matthew 25:29, King James Version)[3].
Finally the Baseline Regime is the "prototypical" scenario upon which we will perform extensive experiments.

4.2 Market selection and birth-death processes

Competitive interactions are captured by a "stochastic quasi-replicator" dynamics:

$$\Delta s_i(t, t-1) = A s_i(t-1)\left(\frac{a_i(t)}{\bar{a}_t} - 1\right) \tag{2}$$

where:

$$\bar{a}_t = \sum_i a_i(t) s_i(t-1) \tag{3}$$

where $s_i(t)$ is the market share of firm i which changes as a function of the ratio of the firm's productivity (or "competitiveness") to the weighted average of the industry. It is a "quasi-replicator" since a genuine replicator lives on the unit simplex. The "quasi" one may well yield negative shares, in which case the firm is declared dead and market shares are accordingly recomputed. Being A an elasticity parameter that captures the intensity of the selection mechanism operated by the market, the death rule implies that whenever it is weak, firms survive irrespectively of their market shares (fitness) and their competitiveness. However, empirically, firms with strikingly low relative competitiveness do die even in environments characterized by low degree of competition. Concerning equation 2 we shall study the effect of different degrees of *market selectiveness*, as captured by the A parameter. In that, note that the *competitive process as such* induces ex-post correlation in growth rates: the growth of the share of any one firm induces the fall of the total share of its complement to one! Finally, entry of new firms occurs proportionally to the number of incumbents present in the market:

$$E(t) = \omega(t) N(t-1) \tag{4}$$

where $E(t)$ is the number of entrants at time t, $N(t-1)$ is the number of incumbents in the previous period and $\omega(t)$ is a random variable uniformly distributed on a finite support (which in the following, for simplicity,we assume drawn from a uniform distribution). The idea that the number of entrants is proportional to the number of incumbents is strongly empirically verified (see e.g Geroski (1991) and Geroski (1995) that finds a significant cross-correlation between entry and exit, but also the preferential attachment scheme in Buldyrev et al. (2007)). The number of firms at each time steps is maintained constant, that is the number of dying is offset by an equal amount of rising firms.
The productivity attributed to the entrants follows the same incumbent rule, according to the Market regime where it takes place, multiplied by the average productivity in the market. What happens is that entrants productivity diverges from the average market productivity of a stochastic component, that is again a random extraction from alternative distributions (Normal, Lognormal, Poisson, Laplace and Beta):

$$a_j(t) = (1 + \theta_j(t)) \sum_i a_i(t) s_i(t-1) \tag{5}$$

where $\theta_j(t)$ is a random variable which parametrizes barriers to learning by entrant, or conversely the advantage of "newness".

[3]For more details see Dosi and Sylos Labini (2007)

4.3 Timeline of the events

- There are N initial incumbent firms. They have at time 0 equal productivity (but in the Schumpeter Mark I regime) and equal market shares .
- At the beginning of each period, if not under regime Mark I, firms learn according to the dynamic of the specified process on productivity.
- Firms acquire or loose market share according to the quasi-replicator.
- Firms exit the market according to the rules of death: $s_i(t) \leq 0$.
- Market shares growth rate are calculated.
- The number of entrants is drawn as a function of the total number of firms at the beginning of the period, and market shares of incumbents are adjusted accordingly.

	Value
Number of firms	150
Number of time steps	500
Number of MC runs	50
Initial productivity	1
Initial market share (1/N)	0.006667
Age Entrants	1
A	1
γ	1
$Beta(\beta_1, \beta_2)$	$[1, 5]$
$Normal(\mu, \sigma)$	$[0.05, 0.8]$
$Lognormal(\mu_1, \sigma_1)$	$[-3.5, 1]$
$Laplace(\alpha_1, \alpha_2)$	$[0.01, 0.015]$
$Uniform(\nu_1\, \nu_2)$	$[0, 0.1]$

Table 1: Parameters initialization.

5 Model properties

Our conjecture is the the replicator dynamics put in act a mechanism of correlation equivalent to the Polya urn mechanism. In particular we can read the stochastic replicator as a Polya urn model. As demonstrated by Schreiber (2001) and discussed in Pemantle (2007), *the stochastic replicator is a generalized Polya urn scheme.* Particularly, based on the description proposed by the latter, at each time $t \geq 0$ there is a population $N(t)$ made by firms whose only attribute is the type i of micro-shocks, being $\{1, ..., i\}$. These firms are represented by an urn with colors $\{1, ..., i\}$, the productivity $a_i(t)$ is the random fitness of firm i and the market share $s_i(t)$ its representation. The size of the population is determined as follows. At each time step t a ball (firm) of color (micro-shock) i is extracted (with replacement) from the urn and returned to that along with $a_i(t)$ extra balls of color i. Being $a_i(t)$ a measure of the fitness of type i in the population, its representation (market share) will change of an amount proportional to its fitness against the others $N(t) - 1$ balls. Repeating this mechanism will allow the growth of type i to be proportional to its own success against all the other types weighted by their own representation in the population, clearly:

$\Delta s_i(t, t-1) = s_i(t-1)a_i(t)/\sum_i a_i(t)s_i(t-1)$. The finite number of opportunities of the Bottazzi and Secchi (2006a) model, reads in our case as the finite, given dimension of the market, that is assumed to be stationary, where the following constrain holds: $\sum_i s_i(t) = 1, \forall t \geq 0$. When the fitness function is not influenced by any random noise, that is, in the *Schumpeter Mark I* regime, the model collapses into a deterministic discrete replicator for the first time step. We will show in the following section that this type of process is able to robustly reproduce fat-tail distribution of growth rates under the three learning regime, and even to generate Laplace distribution under the *Scumpeter Mark II*. In our replicator process, the cumulativeness at the origin of fat-tail, occurs both in space as in Bottazzi and Secchi (2006a) (finite dimension of the market) and in time (autocorrelated productivity).

5.1 Baseline Regime

We will start by analysing the *Baseline Regime* that amount to an intermediate set-up between the two more "extreme" market configurations. Particularly we set:

- The market selection parameter $A = 1$;
- The cumalativiness parameter $\gamma = 0$
- A Beta (1,5) distribution for the extraction of the micro-shocks.

In table 2 the aggregate descriptive statistics, across 500 time steps in the Baseline regime, are presented.

	Average	Min	Max	Sigma
Number of entrants	12.172	0	35	5.72
Average age	8.52	1	12.5	2.06
Average productivity growth	0.046	0.034	0.060	0.004
Average shares growth	-0.088	-0.26	0.016	0.045

Table 2: Descriptive Statistics for 500 time steps. Baseline Regime

In the following the dynamic of productivity and its persistence, size and growth rate distribution, and market turbulence we will presented. In figure 7 the results from the Baseline Regime are reported. Figure 7.a shows the log normalized productivity distribution that reads as:

$$\log n_i(t) = \log a_i(t) - \log \sum_i a_i(t)s_i(t-1) \tag{6}$$

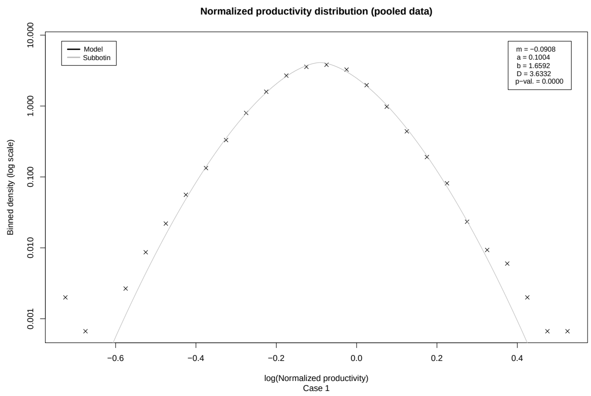
The width of the distribution appears to be rather disperse. Table 3 shows the autocorrelation structure of the (not normalized) productivity distribution across the three regimes. It appears rather strong with a coefficient of $0.97 - 0.98$ at $t - 1$. Proceeding with the analysis of the size distribution, figure 7.b shows the log $rank$−log $size$ plot fitted against a LogNormal distribution. As already discussed, if the size distribution follows a Power Law:

$$sr^{\beta} = A \tag{7}$$

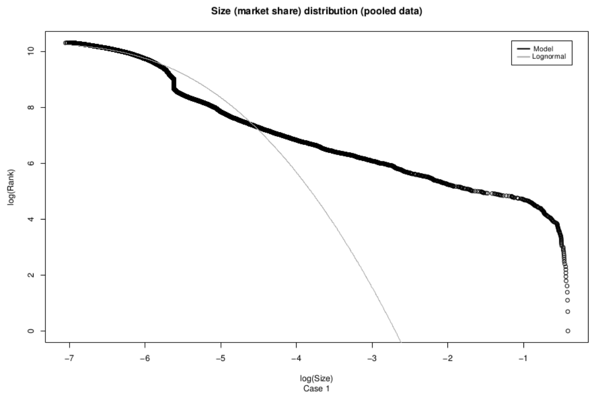
linearising we have:

$$\log r = \alpha + \beta \log s \tag{8}$$

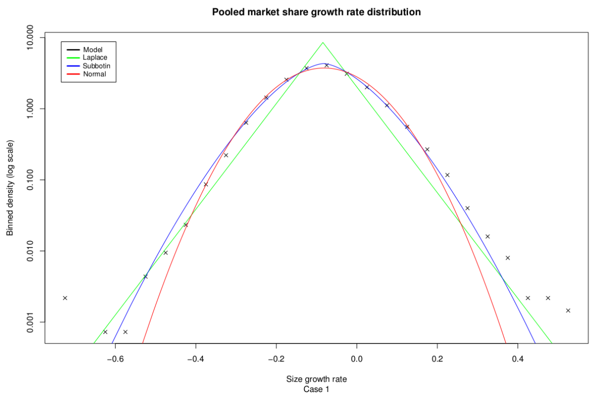
where s and r are respectively the size and the rank of the distribution. β is the slope parameter, and under the Zipf Law (that is a restriction of the Pareto law) it is equal to one. In our case, the slope is clearly different from one, presenting a cut-off point above which



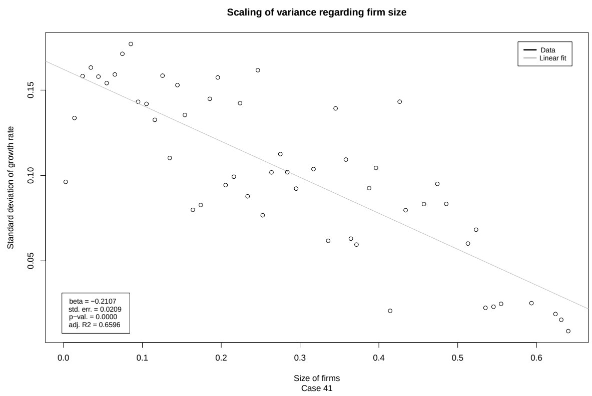
(a) *Normalized productivity distribution.*



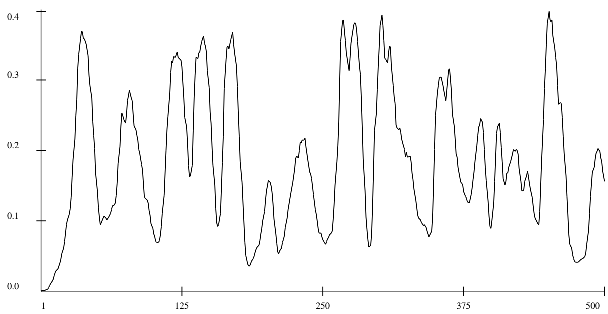
(b) *Pooled size distribution.*



(c) *Growth rates distribution.*



(d) *Scaling variance relation.*



(e) *Herfindahl-Hirschman Index.*

Figure 7: Baseline Scenario

	β	$AR(1)$
Baseline Regime	-0.2562	0.970
	(0.0453)	
Schumpeter Mark I	-0.190	0.986
	(0.0736)	
Schumpeter Mark II	-0.4121	0.987
	(0.058)	

Table 3: Scaling variance relation. Estimation of the slope coefficient across regimes

the Lognormal distribution is not any more a well approximation of the size distribution. Our emphasis, more than to detect the emergence of a Zipf distribution, which as above discussed, it's not robust under sectoral level of disaggregation (and consider our regimes as sectors characterised by a different innovative profile), is devoted to the emergence of the clear-cut skewness and a strong departure from the Lognormal distribution. Finally figure 7.c shows the fat-tail distribution of growth rates, plotting the distribution against a Normal (red line) and a Laplace (green line) one. The growth rate of firm size distribution is defined as:

$$\log g_i(t) = \log s_i(t) - \log s_i(t-1) \tag{9}$$

where market shares represent our proxy for size. It is not necessary to normalize the size by the average growth rate of the market, being the latter equal to zero. In order to understand how fat the tails are, we estimate a symmetric Subbotin function, which is defined by three parameters m, a and b. In particular m is a location parameter, a is a scale parameter and b tells how fat the tails are. The Subbotin is a big family of distributions. According to the value of the parameter b, the Subbotin:

$$f_S(x) = \frac{1}{2ab^{1/b}\Gamma(1/b+1)} e^{-\frac{1}{b}\left|\frac{x-\mu}{a}\right|^b} \tag{10}$$

can yield:

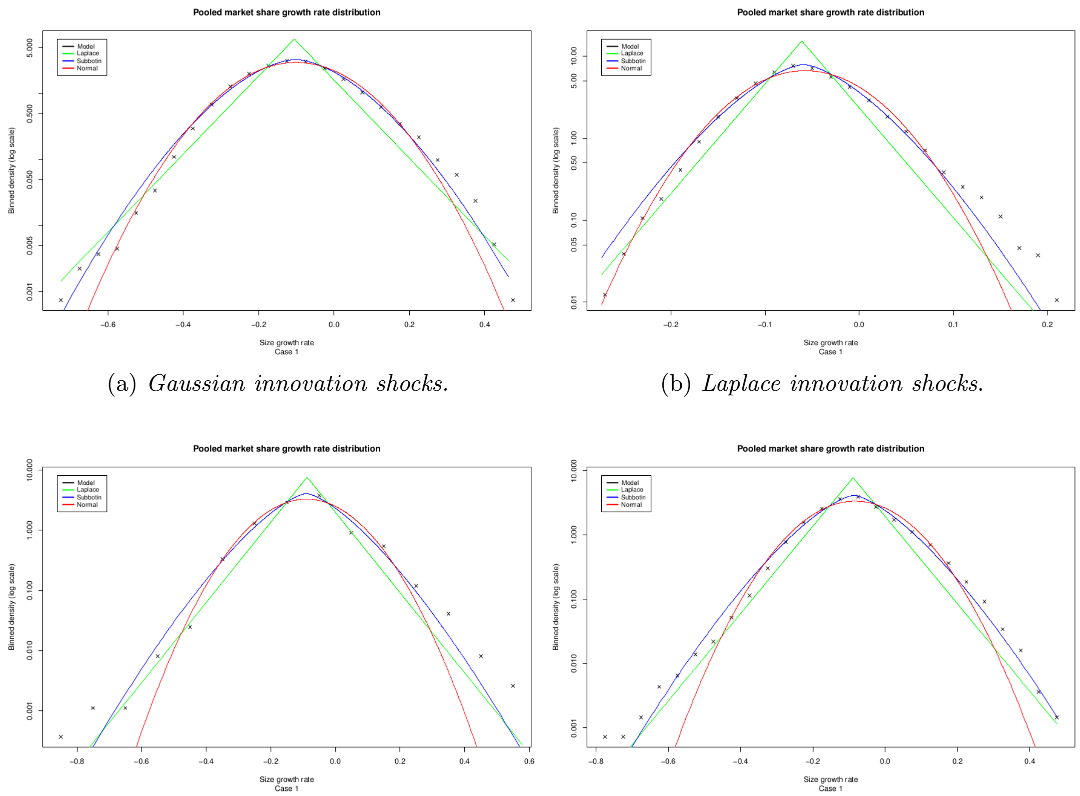
- Gaussian Distribution: $b = 2$
- Laplace Distribution: $b = 1$

The figure shows a rather strong departure from Normality, with the b parameter on average around 1.5, even though, in this baseline configuration we cannot reject the hypothesis of Normality in many cases. The presence of fat-tails in the distribution of growth rates is a sign of the shocks correlation across firms. Correlations among shocks is an hint that my growth erodes yours, that is the selection effect of the replicator dynamic in action. On the contrary, normality means absence of any selection pressure. Figure 7.d shows the negative relationship between the variance of growth rate and size, together with an OLS fit:

$$\sigma(g_i) = \alpha + \beta S_i \tag{11}$$

It is important to underline how, differently from the empirical literature, here we are using shares and not revenues as size proxy. The estimation of the slope coefficient is presented in table 3, together with the R^2 estimation averaged across fifty runs (in brackets the standard deviation). Across the three regimes, the negative relation is always present.

Finally figures 7.e depicts the turbulence of the market showing the dynamics over time of the Herfindahl-Hirschman index. It is rather clear how the market is characterised by persistence fluctuations that are *endogenously* determined by the entry-exit process.



(a) *Gaussian innovation shocks.* (b) *Laplace innovation shocks.*

(c) *Poisson innovation shocks.* (d) *Lognormal innovation shocks*

Figure 8: Baseline Regime. Firm growth rates under different innovation shocks.

We will proceed now with the exploration of the effect of different shapes of the microshock distributions upon the growth rates. Recall that in this baseline regime the innovation process is carried on both by entrants and incumbents. The model exhibits a strong qualitative invariance to the shape of the input distributions of the innovation shocks, as shown in figure 8, confirming the findings by Bottazzi and Secchi (2006a). In table 4 the results of the average parameter values across fifty runs of Monte Carlo simulations (in brackets their standard deviation). The range of the parameter values is between $1.637 - 1.467$. This suggest how the tails of the distribution are very far from being normal.

5.2 Schumpeter Mark I Regime

The Schumpeter Mark I regime where, just to recall, there is no learning process and innovation is carried on by entrants only, is a quite extreme case, that tries to basically isolate the effect of market selection and to test the emergence of fat tail distribution. In table 5 the descriptive statistics are shown. This market it's relatively more calm, with respect to the Baseline Regime, with a lower number of entrants, with more lasting age, and as expected very low shares and productivity growth.

Figure 9.a show the dynamic of productivity and its persistent nature emerge from the autoregressive coefficient already presented in 3. In this particular case, having turned off the extraction of the incumbent growth rates, their initial productivity has been heterogeneously initialized: from a range between $1 - 1.5$, each firm is endowed by an initial amount of productivity that differs from the others. Figure 9.b shows the skew distribution of firm size. It is worthy to note how in this regime an higher fraction of firm size with respect to the baseline regime is characterized by a lognormal distribution. Figure 9.c illustrates the

	m	a	b
Gaussian shocks	-0.102	0.128	1.637
	(0.029)	(0.003)	(0.038)
Laplace shocks	-0.057	0.0566	1.489
	(0.004)	(0.002)	(0.048)
Poisson shocks	-0.0843	0.099	1.327
	(0.0257)	(0.002)	(0.048)
Beta shocks	-0.081	0.096	1.539
	(0.004)	(0.003)	(0.040)
Log-normal shocks	-0.0896	0.1027	1.467
	(0.0039)	(0.0025)	(0.040)

Table 4: Baseline Regime. Parameters estimation across different innovation shocks.

	Average	Min	Max	Sigma
Number of entrants	3.73	0	13	2.09
Average age	22.07	1	30.36	4.14
Average productivity growth	0.000	0	0.37	0.016
Average shares growth	-0.024	-0.11	-0.01	0.01

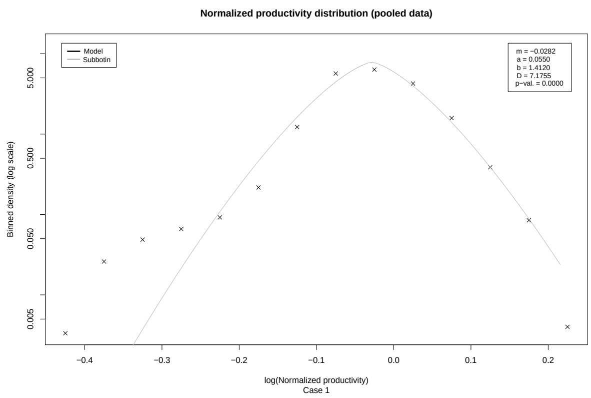
Table 5: Descriptive Statistics for 500 time steps. Schumpeter Mark I Regime

distribution of growth rates. Even in absence of any process of learning, the only effect of market selection operated by the replicator dynamics accompanied by an entry process, is able to determine fat-tail distribution. Particularly, as in the other regime, we test for a possible invariant property across distributions. Figure 10 represents the firm growth rates wherein the entrants productivity is extracted from different distribution. Also in this case the persistent nature of fat tails is presented in table 6 where the maximum value of the b parameter is recorded for Normal shocks up to the 0.9 value of the Poisson distributed shocks. Finally in 9.d the dynamic of the HHI index that is relatively calmer respect to the Baseline Regime.

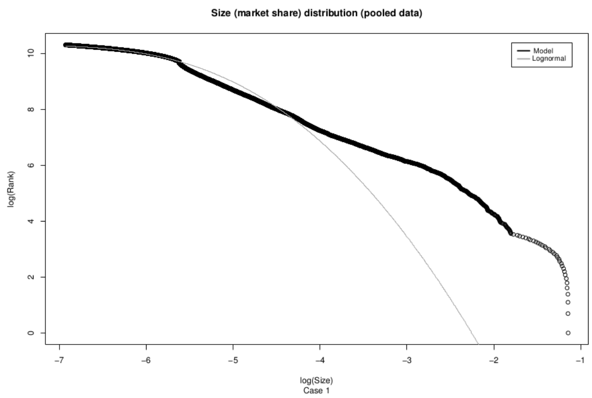
5.3 Schumpeter Mark II Regime

Finally we shall explore a purely cumulative regime in the learning process. In table 7 the aggregate descriptive statistics. Contrary to the Mark I, in this Regime we have an higher degree of turbulence, with many entrants in the market, with an average age of five periods (relatively close to the empirical recorded values).

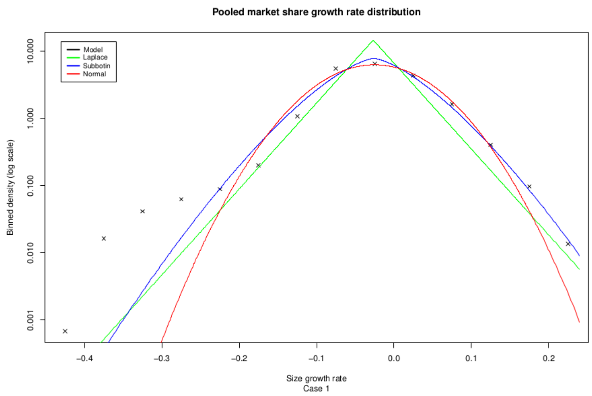
In figure 11 the dynamics of productivity, size and market shares are presented. As in the baseline regime the persistent heterogeneity of productivity is shown in figures 11.a. The skewness of the size distribution and the fat-tail nature of growth rates are illustrated in figures 11.b and 11.c. The scaling variance relation in 11.d. In figure 11.e the behaviour of the concentration index concludes the description of the cumulative regime. Compared to the baseline scenario, the distribution of the growth rate in the Schumpeter Mark II manifests a closer shape to the Laplace, with a b parameter on average equal to 1.3. This suggests that an higher cumulativeness in the learning process hence an higher memory of the past performances, increases the autocorrelation in time, and make the shape of the



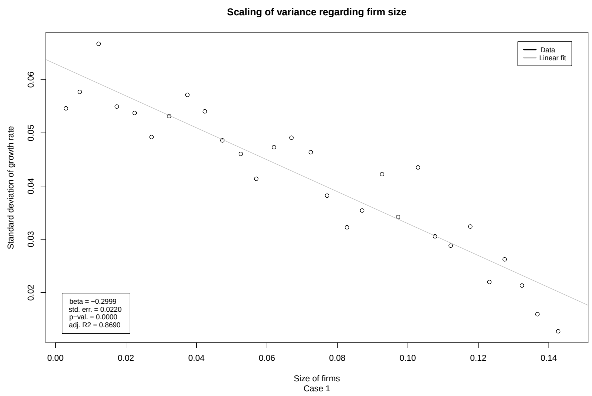
(a) *Normalized productivity distribution.*



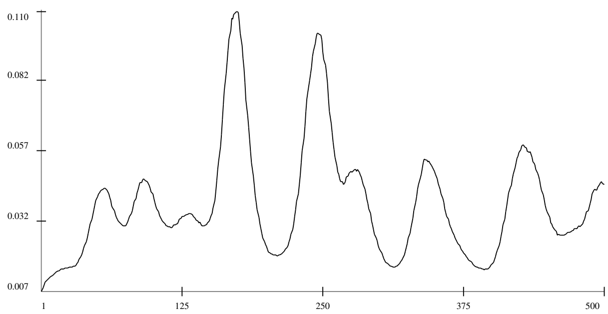
(b) *Pooled size distribution.*



(c) *Growth rates distribution.*

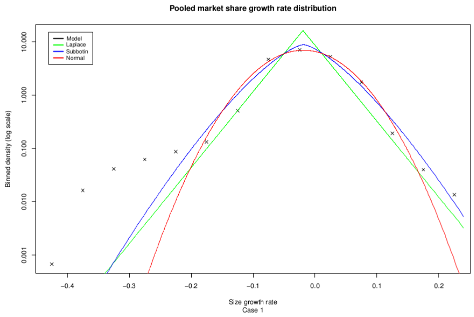


(d) *Scaling variance relation.*

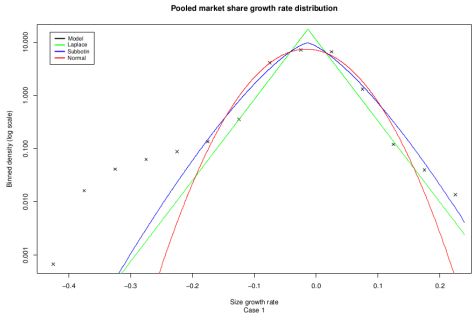


(e) *Herfindahl-Hirschman Index.*

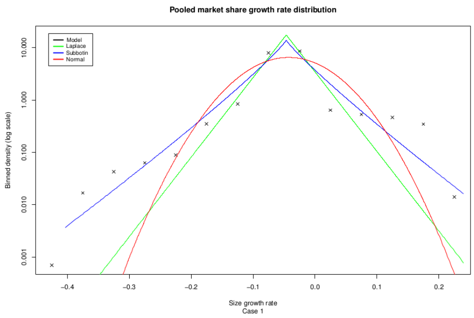
Figure 9: Schumpeter Mark I Regime



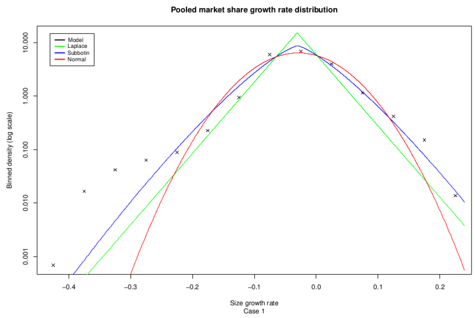
(a) *Gaussian innovation shocks.*



(b) *Laplace innovation shocks.*



(c) *Poisson innovation shocks.*



(d) *Lognormal innovation shocks*

Figure 10: Schumpeter Mark I. Firm growth rates under different innovation shocks

	m	a	b
Gaussian shocks	-0.0198	0.0502	1.426
	(0.00066)	(0.0011)	(0.026)
Laplace shocks	-0.013	0.045	1.360
	(0.0018)	(0.0012)	(0.027)
Poisson shocks	-0.046	0.0389	0.900
	(0.0008)	(0.0013)	(0.0056)
Beta shocks	-0.025	0.053	1.397
	(0.001)	(0.001)	(0.023)
Log-normal shocks	-0.0307	0.0543	1.329
	(0.001)	(0.0013)	(0.020)

Table 6: Scumpeter Mark I. Parameters estimation across different innovation shocks.

	Average	Min	Max	Sigma
Number of entrants	17.4	0	67	9.6
Average age	5.3	1	10.43	1.79
Average productivity growth	0.039	0.013	0.060	0.006
Average shares growth	-0.13	-0.49	0.032	0.083

Table 7: Descriptive Statistics for 500 time steps. Schumpeter Mark II Regime

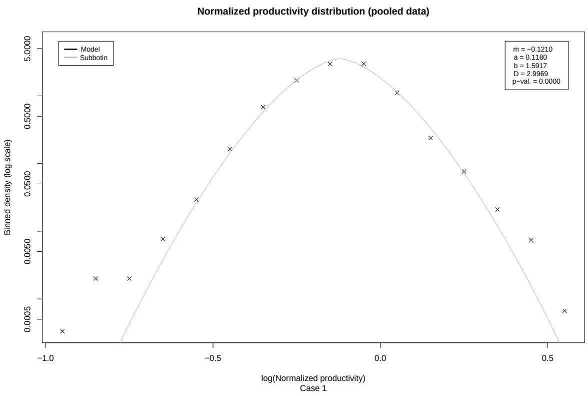
	m	a	b
Gaussian shocks	-0.162	0.147	1.402
	(0.00543)	(0.007)	(0.053)
Laplace shocks	-0.087	0.068	1.284
	(0.0038)	(0.0043)	(0.049)
Poisson shocks	-0.096	0.1008	1.231
	(0.0039)	(0.0026)	(0.0523)
Beta shocks	-0.113	0.1074	1.367
	(0.0047)	(0.0046)	(0.0409)
Log-normal shocks	-0.118	0.1132	1.327
	(0.0060)	(0.0046)	(0.045)

Table 8: Schumpeter Mark II Regime. Parameters estimation across different innovation shocks.

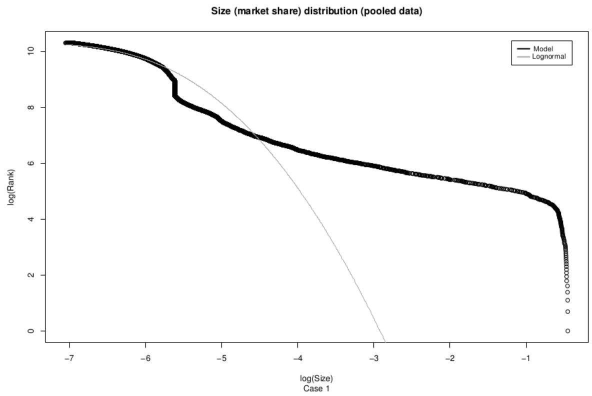
growth rates more Laplacian. Figure 12 shows the invariant shape of the growth rates under different innovation shock distributions. Table 8 reports the estimated parameter values and the relative standard deviation.

5.4 Cumulativeness and selection

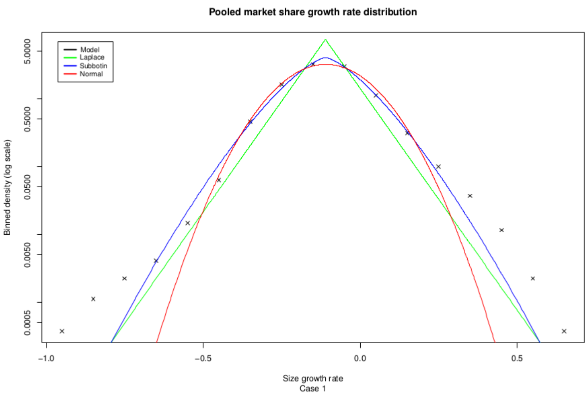
In the Schumpeter Mark II regime we explore the effects on the distribution of firms growth rates of two different parameters: the γ parameter which captures the degree of cumulativeness in the learning process and the A parameter which embodies the degree of selectivity in the market. The default distribution of the innovation shocks is a usual a Beta(1,5). We start analysing the effect of cumulativeness. As expected, the increase in the γ parameter, shown in figure 13 induces a more tent-shaped distribution in the growth rates up to becoming "super Laplacian" (see 13.c). Table 9 shows the negative relation between the γ and the b parameters. This result is extremely important: the empirical distribution of the firm growth rates has been extensively proven to be tent-shaped. Actually, the Schumpeter Mark II appears to be the regime that best replicates the empirical growth rates distribution. This is equivalent to say that in the real markets a strong "Matthew effect" in the accumulation of capabilities takes place. It is worthy to underline that the transmission mechanism start from the cumulative learning process and affects selection.
Regarding the effect of the A parameter, it is rather clear how a low selection pressure allows firms with lower growth rates to survive, moving the mass of the distribution on the lower part of the support (see figures 15.a and 15.b), where the high growth firms, the "gazzellas" occupies the upper-left tail. Conversely, when the selection pressure increases (see 15.c and 15.d), the more inefficient firms are frozen out by the market, the mass of the distribution shifts in the left-part, whit just few alive inefficient firms that occupy the left tail. We



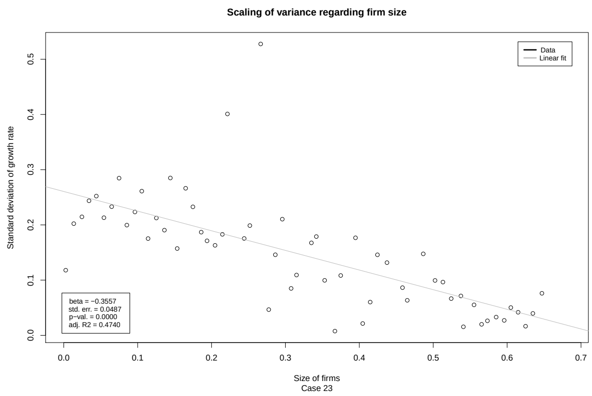
(a) *Normalized Productivity distribution*



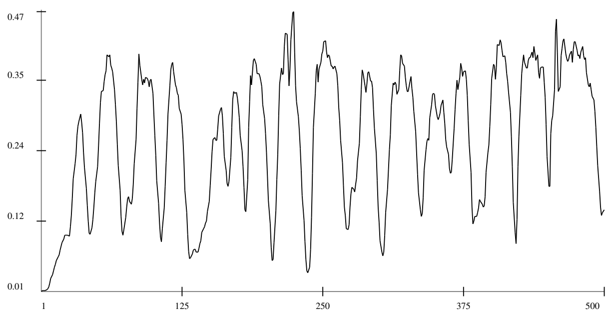
(b) *Pooled size distribution*



(c) *Growth rates distribution*

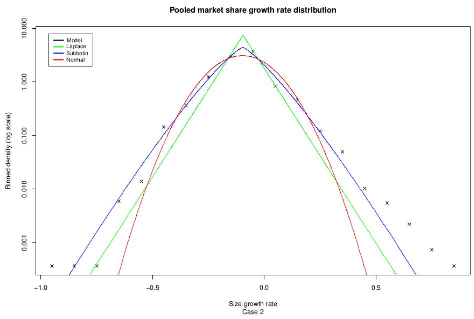


(d) *Scaling variance relation.*

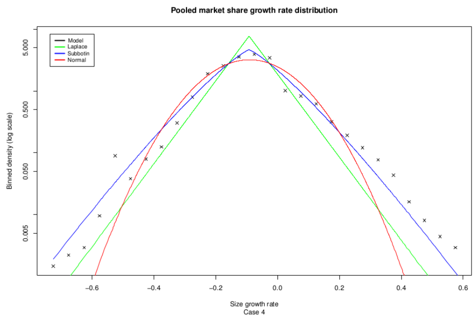


(e) *Herfindahl-Hirschman Index.*

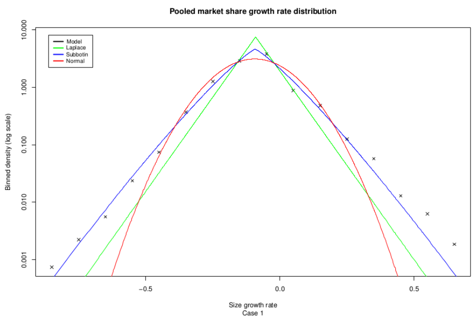
Figure 11: Schumpeter Mark II Regime



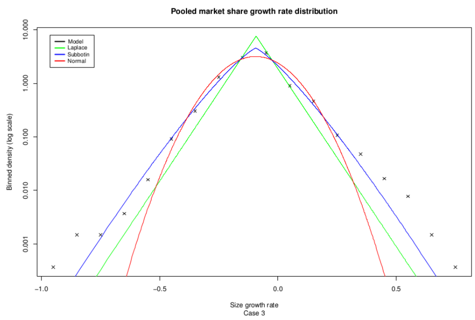
(a) *Gaussian innovation shocks.*



(b) *Laplace innovation shocks.*



(c) *Poisson innovation shocks.*



(d) *Lognormal innovation shocks*

Figure 12: Schumpeter Mark II Regime. Firm growth rates under different innovation shocks.

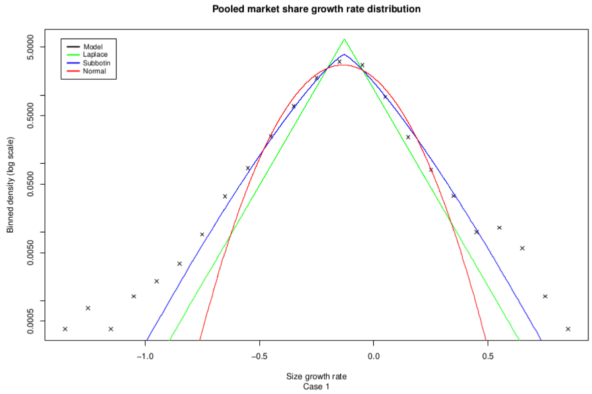
	b
$\gamma = 1.5$	1.293
	(0.0568)
$\gamma = 2$	1.176
	(0.0797)
$\gamma = 3$	0.811
	(0.100)

Table 9: Schumpeter Mark II Regime. The effect of cumulativeness

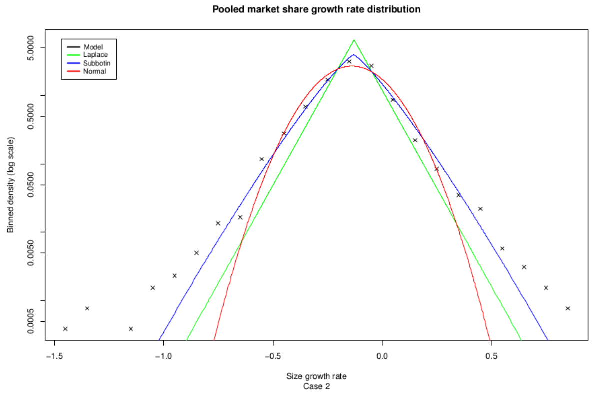
then deduce that the selection parameter is responsible for the symmetry of the distribution. What happens to the the productivity distribution under a low selection pressure? Figure 14 shows how, under a low selectivity, the support of the distribution increases, becoming closer to the empirical ones and a more asymmetric distribution of productivity, with a thicker left tail emerges. This means that, given the same conditions on productivity, the low degree of selection, allows to a big portion of low productive firms to operate in the market.

6 Conclusions

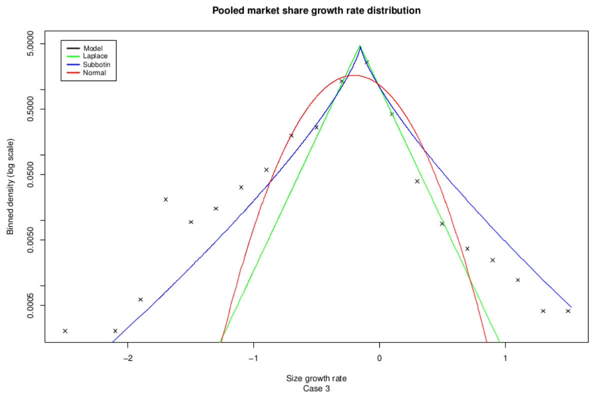
Empirically one ubiquitously observes a large ensemble of micro-stylised facts. In this paper we address the possible causes of these phenomena. Here we investigate what kind of economic process can generate fat tail distribution of the firm growth rates, together with persistent heterogeneity in productivity, skewness in firm size distributions negatively related with the growth rate standard deviation. In particular we focus on a bare-bone evolutionary



(a) $\gamma = 1.5$

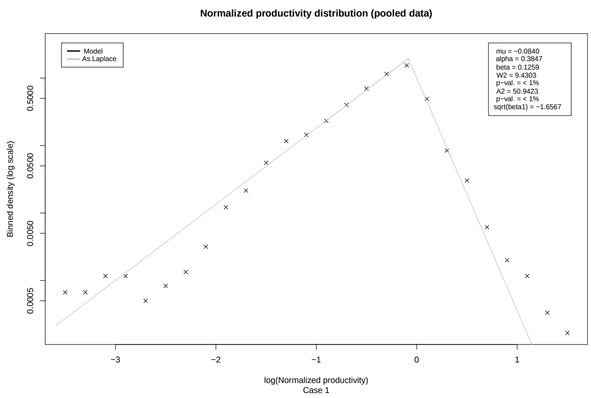


(b) $\gamma = 2$.



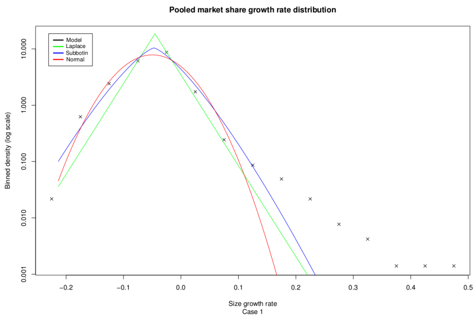
(c) $\gamma = 3$.

Figure 13: Schumpeter Mark II Regime. Firm growth rates under different cumulativeness.

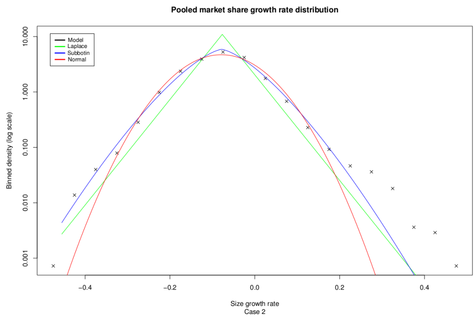


(a) *Normalized productivity distribution under low selection pressure.*

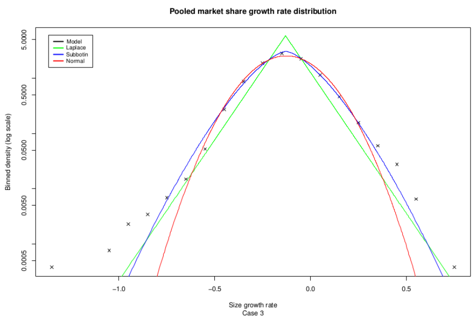
Figure 14: Schumpeter Mark II Regime. $A = 0.2$



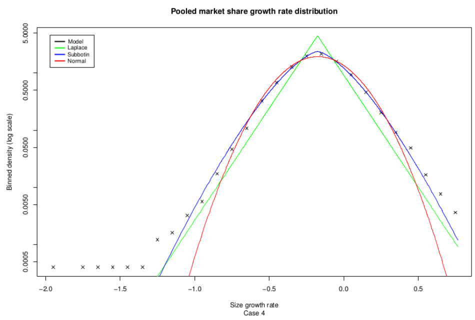
(a) A=0.2



(b) A=0.5



(c) A=1.5



(d) A=2

Figure 15: Schumpeter Mark II Regime. Firm growth rates under different selection pressure.

	m	a	b
A=0.2	-0.0478	0.044	1.423
	(0.00194)	(0.00192)	(0.0697)
A=0.5	- 0.0760	0.069	1.355
	(0.00316)	(0.0030)	(0.0540)
A=1.5	-0.144	0.1412	1.419
	(0.0063)	(0.00621)	(0.048)
A=2	-0.174	0.1758	1.458
	(0.0085)	(0.0064)	(0.0289)

Table 10: Schumpeter Mark II Regime. The effect of different selection pressure

model where the two pillars of evolution, namely, learning and market selection, interact. We examine three alternative market regimes: a *Schumpeter Mark I*, wherein no learning for incumbents take place, an *intermediate* regime where incumbents do learn, and a *Schumpeter Mark II* where the learning process is cumulative. The learning regimes interacted with a "market regime" captured by some form of replicator dynamic. The quite remarkable finding is that under all regimes competitive interactions induces correlation in the growth dynamics of firms and thus the absence everywhere of a normal distribution of growth rates. Additionally, persistent heterogeneity across firms and skewed size distribution are recovered by the model. Fat tails emerge everywhere together with the scaling variance relation. Moreover with cumulative learning the distributions of growth rates turn out to be Laplacian.
However even under the Schumpeter I regime the very process of competitive selection generates fat tails, also in absence of any learning.

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