The Eurace@Unibi Model: An Agent-Based Macroeconomic Model for Economic Policy Analysis

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1 Introduction

This document provides a description of the modeling assumptions and economic features of the Eurace@Unibi model. Furthermore, the document shows typical patterns of the output generated by this model and compares it to empirically observable stylized facts. The Eurace@Unibi model provides a representation of a closed macroeconomic model with spatial structure. The main objective is to provide a micro-founded macroeconomic model that can be used as a unified framework for policy analysis in different economic policy areas and for the examination of generic macroeconomic research questions. In spite of this general agenda the model has been constructed with certain specific research questions in mind and therefore certain parts of the model, e.g. the mechanisms driving technological change, have been worked out in more detail than others.

The purpose of this document is to give an overview over the model itself and its features rather than discussing how insights into particular economic issues can be obtained using the Eurace@Unibi model. The model has been designed as a framework for economic analysis in various domains of economics. A number of economic issues have been examined using (prior versions of) the model (see Dawid et al. (2008), Dawid et al. (2009), Dawid et al. (2011a), Dawid and Harting (2011), van der Hoog and Deissenberg (2011), Cincotti et al. (2010)) and recent extensions of the model have substantially extended its applicability in various economic policy domains, however results of such policy analyses will be reported elsewhere. Whereas the overall modeling approach, the different modeling choices and the economic rationale behind these choices is discussed in some detail in this document, no detailed description of the implementation is given. Such a detailed documentation is provided in the accompanying document Dawid et al. (2011b).

Agent-based simulation models are a relatively new addition to the tool-box of economists, and a growing group of economic scholars sees them as a useful instrument complementing established modeling approaches. The potential merits and the challenges of the use of an agent-based approach for economic analysis in general and the examination of economic policy issues in particular have been discussed for example in LeBaron and Tesfatsion (2008), Farmer and Foley (2009), Dawid and Neugart (2011) or Fagiolo and Roventini (2011). Closed macroeconomic models that follow an agent-based approach and are in spirit related to the Eurace@Unibi model have been developed among others by Delli Gatti et al. (2008), Dosi et al. (2010) or Mandel et al. (2010). The Eurace@Unibi model differs from each of these in various aspects. Important distinguishing features of the Eurace@Unibi model are its spatial structure, the systematic and empirical foundations of the behavioral rules of (firm) agents implied by the 'Management-Science approach', the asynchronous decision making of individuals, the systematic use of a balance-sheet approach making all flows in the modeled economy transparent and ensuring stock-flow consistency. Furthermore, Eurace@Unibi is characterized by a detailed description of crucial aspects of the dynamic interplay between labor markets (skill dynamics, (spatial) labor flows), industry evolution (productivity change, technology diffusion), credit markets (credit rationing, total debt dynamics) and consumption. In several of these domains new approaches have been developed to capture important economic mechanisms in an agent-based framework. Examples in this respect are the determination of pricing behavior of firms, the worker-employee matching and determination of wages in the presence of workers that are heterogenous with respect to some observable and some unobservable characteristics, the endogenization of the speed of technology diffusion under explicit consideration of firms' technology choices or the explicit consideration of the (empirically substantiated) complementarity between workers' skills and physical capital quality in production.

The Eurace@Unibi model presented here is based on the agent-based macroeconomic simulation platform developed within the EURACE project. This EU-funded project (EU IST FP6 STREP grant 035086) was carried out by a consortium lead by S. Cincotti (University of Genova), H. Dawid (University of Bielefeld), C. Deissenberg (Université de la Mediterrané), K. Erkan (TUBITAK National Research Institute of Electronics and Cryptology), M. Gallegati (Università Politecnica delle Marche), M. Holcombe (University of Sheffield), M. Marchesi (Università di Cagliari), C. Greenough (STFC - Rutherford Appleton Laboratory). After the completion of the EURACE project in 2009 the authors of this manuscript have extended and altered the model substantially in numerous directions leading to the current version of the model. In particular, substantial changes and extensions have been made to the production technology, to the model of the decision processes of consumption good producers, to the investment good production sector and to the bancrupcy procedures of firms. Furthermore, interest payments of banks have been introduced on the credit market and the structure of the balance sheets has been completely overhauled. Finally, the financial market of the original EURACE model has been replaced by a much simpler model following a slightly different approach than the original one. At the end of this document we provide a detailed list of items where the present model differs from the EURACE model from 2009.

2 Main Features and Overall Structure

2.1 Overall Structure

The model describes an economy containing labor, consumption goods, capital goods, financial and credit markets in a regional context. The economy is inhabited by numerous instances of different types of agents: firms (consumption goods producers and capital goods producers), households and banks in the economy. Each of these agents is located in one of the regions. Additionally, there is a single central bank and a government that collects taxes and finances social benefits as well as potentially some economic policy measures, where policies might differ between regions. Finally, there is a statistical office (Eurostat) that collects data from all individual agents in the economy and generates aggregate indicators according to standard procedures. These indicators are distributed to the agents in the economy (which might use them e.g. as input for their decision rules) and also stored in order to faciliate the analysis of the simulation results.

Capital goods of different quality are provided by capital goods producers with infinite supply. The technological frontier (i.e. the quality of the best currently available capital good) improves over time, where technological change is driven by a stochastic (innovation) process. Firms in the consumption goods sector use capital goods combined with labor input to produce consumption goods. The labor market is populated with workers that have a finite number of general skill levels and acquire specific skills on the- job, which they need to fully exploit the technological advantages of the capital employed in the production process. Every time when consumption goods producers invest in new capital goods they decide which quality of capital goods to select, thereby determining the speed by which new technologies spread in the economy. Consumption goods are sold at local market platforms (called malls), where firms store and offer their products and consumers come to buy goods at posted prices. Labor market interaction is described by a simple multi-round search-and-matching procedure where firms post vacancies, searching workers apply, firms make offers and workers accept/reject. Wages of workers are determined, on the one hand, by the expectation the employer has at the time of hiring about the level of specific skills of the worker, and, on the other hand, by a base wage variable, which

is influenced by the (past) tightness of the labor market and determines the overall level of wages paid by a particular employer. Banks collect deposits from households and firms and give credits to firms, where the interest firms have to pay on the amount of their loan depends on the financial situation of the firm and the amount of the loan might be restricted by the bank's liquidity and risk exposure. There is a financial market where shares of single asset are traded, namely an index bond containing all firms in the economy. The dividend paid by each share at a certain point in time is determined by the sum of the dividends currently paid by all firms. This simple representation of a financial market is not suitable to describe speculative bubbles in the financial market, but captures important feedbacks between firm profits and households income, in a sense that fluctuations of dividends affect only the income of a particular subgroup of households, namely the owners of shares of the index bonds. The central bank provides standing facilities for the banks at a given base rate, pays interest on banks' overnight deposits and might provide fiat money to the government.

The spatial extensions of the markets differ. The capital goods market is global meaning that firms in all regions buy from the same global capital good producer and therefore have access to the same technologies. On the consumption goods market demand is determined locally in the sense that all consumers buy at the local mall located in their region, but supply is global because every firm might sell its products in all regional markets of the economy. Labor markets are characterized by spatial frictions determined by commuting costs that arise if workers accept jobs outside their own region. It is assumed that households and firms have access to all banks in the economy and therefore credit markets operate globally.

2.2 Timing

The basic time unit in the model is one day. Overall there are calendar driven and event-based activities. Many decisions, like production choice or hiring of firms, are taken monthly (each month has 20 (working) days) and others, like consumption decisions of households are taken weekly (each week has 5 (working) days). Each agent has a particular day to act once a year/month/week for each decision, where these days to act might differ between agents of the same type. Therefore, there is in general no synchronization between different actors, although for each type of decision or action the time spans between subsequent days to act are identical for all firms respectively households. It is well known that artificial overshooting effects might arise, if decisions, like output determination or pricing, are always taken simultaneously by all individuals, and therefore this type of synchronization is avoided here. It should however be noted that certain activities, like tax collection, are synchronized due to institutional reasons. Only few activities in the model are event-based, like the application for unemployment benefits at the point in time when an employee becomes unemployed.

2.3 Decision Making

In contrast to dynamic equilibrium models, where it is assumed that the behavior of all actors is determined by maximization of the own (inter-temporal) objective function using correct expectations about the behavior of the other actors, agent-based simulation models need to provide explicit constructive rules that describe how different agents take different decisions. Actually, the need to provide such rules is not only based on the basic conviction underlying these models, that in most economic settings actual behavior of decision makers is far apart from inter-temporally optimal behavior under rational expectations, but also on the fact that in most models that incorporate heterogeneity among agents and explicit interaction protocols (e.g. market rules) the characterization of dynamic equilibria is outside the scope of analytical

and numerical analysis. Given that need to specify explicit rules for all decisions taken by all actors in an agent-based model the determination and motivation of the implemented rules becomes a major modeling issue. The 'Wilderness of bounded rationality' (Sims (1980)) is a serious concern since a large number of different approaches to model boundedly rational behavior and its adaptation have been put forward in the literature and at this point there is little indication for the emergence of a widely accepted consensus that provides empirically or theoretically well founded concepts for tackling this issue.

The choice of the decision rules in the Eurace@Unibi model is based on a systematic attempt to incorporate rules that resemble empirically observable behavior documented in the relevant literature. Concerning households, this means that for example empirically identified saving rules are used and purchasing choices are described using models from the Marketing literature with strong empirical support. With respect to firm behavior we follow the 'Management Science Approach', which aims at implementing relatively simple decision rules that match standard procedures of real world firms as described in the corresponding management literature. There is a rich literature on (heuristic) managerial decision rules in many areas of management science. This includes pricing (see e.g. Nagle and Hogan (2006)), production planning (see e.g. Silver et al. (1998)) or market selection (see e.g. Wind and Mahajan (1981), Kotler and Keller (2009)). Although, it certainly cannot be assumed that all firms in an economy rely on such standard managerial heuristics, capturing the main features of these heuristics when modeling the firm adds a strong empirical micro foundation to the agent-based model. Generally speaking, this approach implies that a parametrized decision rule is developed for each decision each agent-type has to take. Whereas the parameter values determining the exact form of the rule might differ between individuals and might change over time, it is assumed that all individuals share the same (empirically motivated) structure for each type of rule and that this structure also does not change over time. The concrete implications of the Management Science approach will be illustrated in the following Section, where the different parts of the model are desribed in more detail. A more extensive discussion of the Management Science approach can be found in Dawid and Harting (2011).

2.4 Heterogeneity and Aggregation

A main feature of agent-based models is their ability to explicitly account for different types of heterogeneities between individuals and to study the effects of such heterogeneities on economic dynamics, effects of economic policy etc. The Eurace@Unibi framework allows for the consideration of different types of heterogeneity:

- *Persistent Heterogeneity within Regions:* Attributes that describe certain characteristics of agents, that do not change over time, might vary between individuals. In such cases parameters of individuals are determined stochastically according to distributions provided by the model user. Examples of such parameters are general skills of individual workers.
- *Persistent Heterogeneity between Regions:* The distributions according to which agent-parameters are determined might differ between regions in order to describe structural differences between regions. Furthermore, policy measures might differ between regions.
- *Initial Heterogeneity (within and between Regions):* Variables, that are adjusted endogeneously during the simulation run, might be initialized differently for different agents. The distributions according to which these initial values are drawn might differ between regions in order to capture the effects of different starting conditions of economies on economic dynamics and effects of certain policy measures.

- *Emergent Heterogeneity:* Typically most of the dynamic variables evolve differently for different individual agents and heterogeneity emerges, even if individuals are initialized homogeneously. The developed framework allows to capture the evolution of the population distribution of the relevant variables both by using statistical indicators and by visualizing the dynamics of a certain variable for all individuals in a set of agents.

Aggregation of individual data is done by the Eurostat agent and follows in principle the procedures used by real world statistical offices.

2.5 Implementation Issues

The Eurace@Unibi model has been implemented in the *Flexible Large-scale Agent-based Modelling Environment* FLAME (see http://www.flame.ac.uk). Several features make this environment particularly appealing as a framework for the analysis of large scale agent-based economic models. It has been particularly designed for use on parallel computers, which is a big advantage when large agent-based models are simulated. Also FLAME provides a very transparent and clean way to model information flows between agents. The only means to communicate private data between agents are messages, where the data an agent can transmit using a message consists of a list of values of its own state variables (e.g. wealth, income, skills, profits, expectations about certain variables). Messages are added to a centralized message board and the sender determines which agents can read the message. Agents check the message boards daily in order to collect all the information they are supposed to get. Each agent then can use the collected information as input to decision rules or as the basis for updating some own state variables.

Data generation and analysis is a multi-stage process in which considerations of computation time and data storage play an important role. At the simulation design stage (before simulations are actually run), the model analyst can select to output either a complete snapshot in which all variables of all the agents are stored (very data intensive), or to select a subset of agents (for which all variables will be stored). In addition, it is also possible to select a certain frequency at which to output the data, say every n iterates.

In the simulation stage the data is first stored in XML tagged data files and then transformed into SQL databases. The post-processing is done using the R statistics package in which scripts have been developed for post-processing the data to automatically generate full sets of graphs (timeseries, boxplots, scatter plots, histograms). The plots are generated at two levels: aggregate plots for macro data and disaggregated plots for individual agent data. Finally, the full sets of graphs are available to analyze single runs, batch runs (multiple runs) and for parameter sensitivity analysis.

3 The Model

3.1 The sequence of activities

Agent actions can be time-driven or event-based, where the former can follow either subjective or objective time schedules. Furthermore, the economic activities take place on a hierarchy of time-scales: yearly, monthly, weekly and daily activities all take place following calender-time or subjective agent-time. Agents are activated asynchronously according to their subjective time schedules that is anchored on an individual activation day. These activation days are uniformly randomly distributed among the agents at the start of the simulation, but may change endogenously (e.g., when a household gets re-employed, its subjective month gets synchronized with the activation day of its employer due to wage payments).

The branching of agent activities provides the agents with a high degree of autonomy to initiate their actions independently of the actions of any other agent or central mechanism.

From the model point of view, the mixture of event-based and time-driven actions leads to asynchronous behavior, incomplete information and a mechanism to randomly match the agents on the markets. As such it provides us with a necessary ingredient to model a decentralized market economy.

Each firm proceeds through the following sequence of economic activities:

1. On the firm's ideosyncratic activation day the firm starts its production cycle with production planning. The production plan consists of planned output based on historical observations and the results of market research. Based on the production plan the firm determines its planned input demand for capital and labour.

2. Financial management of the firm. The firm computes the costs of production and the costs for financing its commitments. If the internal resources are insufficient the firm tries to finance externally by requesting credit.

3. Credit market with direct firm bank linkages opens. The banks provide credit by servicing the loan request on a first come first serve basis. The bank decides on the credit conditions for the applying firm (interest rate and amount of credit provided) depending on the firm's financial situation. If the credit request is refused, or not fully accepted, the firm has to reduce its planned production quantity.

4. Bankruptcy of two types. If the firm is credit constrained to such extent that it is not able to pay the financial commitments it becomes iliquid and illiquidity bankruptcy is declared. If at the end of the production cycle revenues are so low that the firm has negative net worth, the firm is insolvent and insolvency bankruptcy is declared. In both cases it goes out of business, stops all productive activities and all employees loose their jobs. The firm writes off a fraction of its debt with all banks with which it has a loan and stays idle for a certain period before it becomes active again.

Table 1: Main agents, markets, and messages in the model.

Agent	Context	Role	Messages
Household	Consumption goods market	Buyer	units demanded
	Labour market	Worker	application, accept/reject job
	Credit market	Depositor	cash holdings
	Financial market	Investor	index share orders
Firm	Investment goods market	Buyer	units demanded
	Consumption goods market	Seller	price, quality
	Labour market	Employer	vacancy, job offer
	Credit market	Borrower	loan request
Investment Goods Firm	Investment goods market	Seller	price, productivity
	Labour market	Employer	vacancy, job offer
Bank	Credit market	Lender	credit conditions
Government	Public sector		tax payments
Central Bank	Credit market	Regulator	base interest

5. Capital goods market. Depending on the amount of financing secured by the firm, it makes physical capital investments. This consists of a vintage choice where the productivity of the capital stock is updated if new vintages are installed.

6. A decentralized labour market opens where firms with open vacancies are matched to unemployed workers. The matching is based on the firm's wage offer and on the worker's skill level and reservation wage.

7. Production takes place on the firm's activation day. After production is completed, the output is distributed to local malls. Firms offer goods at posted prices with price revisions occuring once a year.

8. At the end of the production cycle the firm computes its revenues, and updates its income statement and balance sheet. It pays taxes, dividends, interests and debt installments. It checks if net worth is negative and if so, declares bankruptcy. Otherwise it continues with the next production cycle.

The households' activity sequence is given by:

1. Households receive labour income on the same day that the firm starts its production cycle (wages are paid at the start of the month). Dividend income on index shares is received on the first day of the calender month.

2. After tax payment, households determine their consumption budget for the upcoming month, based on a target wealth to income ratio. Since wealth consists of liquid monetary assets and shares, the consumption decision affects the (planned) asset portfolio. The household first enters the financial market before the consumption goods market opens.

3. Financial market transactions between households consists of trades in index shares. The possibility of rationing and the resulting asset allocation may result in adjustment of the consumption budget.

4. Households go shopping on a weekly basis, selecting among consumption goods offered at their local mall. The consumer tries to spend its monthly budget equally over four weeks. Parts of the weekly budget that are not spent in a given week are rolled over to the consumption budget for the following week.

5. A households that becomes unemployed enters the labour market and receives unemployment benefits.

In the following sections we describe these activities in detail. A full description of the C code is available online as a User Manual (Dawid et al., 2011b).[1]

[1] http://www.wiwi.uni-bielefeld.de/vpl1/research/eurace-unibi.html

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Yearly:
- determine retail price

Monthly:
- production planning: decide planned output
- determine input demand for capital and labour
- financial management
- credit market interaction
- capital goods market interaction
- labour market interaction
- production of output and distribution to malls
- accounting: revenues, income statement and balance sheet
- pay taxes, dividends

Event-based:
- bankruptcy protocol: entry and exit (if equity<0)
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Listing 1: Firm sequence of economic activities.

The households' activity sequence is given by:

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Monthly:
- receive labour income (unemployment benefits if unemployed)
- receive capital income (dividends)
- pay taxes
- consumption/savings decision for upcoming month
- financial market trading

Weekly:
- consumption goods market interaction

Event-based:
- labour market interaction (if unemployed)
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Listing 2: Household sequence of economic activities.

3.2 Consumption goods producer

3.2.1 The Quantity Choice

The consumption goods market in our model consists of local malls that are serviced by all consumption goods producing firms to offer their products and visited by all households located in that region for their shopping activities. At these malls each consumption goods producer holds an inventory that is replenished once a month. The delivery volume of an individual firm in each month is the difference between a planned stock and the current inventory level. As will become clear from the descirption of consumer choice behavior (see Section 3.6.4) actual demand for the product of a firm in a given mall and a given month is stochastic and there are stock-out costs, because consumers intending to buy the product of a firm will move on to buy from a different producer in case the firm's stock at the mall is empty. Therefore, the firm faces a production planning problem with stochastic demand and stock-out cost. The production planning literature has extensively dealt with problems of this type and there are numerous heurisitc decisions rules available in the corresponding managerial literature. The simplest standard heuristic prescribes to generate an estimation of the distribution of demand and then choose the planned stock level after delivery such that the (estimated) stock-out probability during the following month equals a given parameter value (which is influenced by stock-out costs, inventory costs and risk attitude of the firm (see e.g. Silver et al. (1998)). Following our general Management Science approach for modelling firm behavior, a heuristic of this type is adopted by the consumption good producers in the model.

We assume that the demand generated by households visiting these malls during a month can be estimated according to

$$\hat{D}_{i,r,t}(p^*) = \hat{D}_{i,t}(p^*) \cdot \frac{\hat{S}_{r,t}}{\hat{S}_t^{Tot}}. \tag{1}$$

Here, $\hat{S}_{r,t}$ is the estimated market size of the local market r (the real consumption budget of inhabitants of region r) that is ascertained together with the total market size $\hat{S}_t^{Tot}$. The local demand estimation in a mall is the relative share of the estimated total demand. $\hat{S}_{r,t}$, $\hat{S}_t^{Tot}$, and the total demand depending on the current price p^*, $\hat{D}_{i,t}(p^*)$, are estimated on a yearly base within an elaborated market research procedure. For more details regarding the market research activities the reader is referred to Section 3.2.7.

The determination of the planned delivery volumes $\tilde{D}_{i,r,t}$ to each mall is the difference between a critical inventory stock $Y_{i,r,t}$ and the current mall stock $SL_{i,r,t}$,

$$\tilde{D}_{i,r,t} = \begin{cases} 0 & SL_{i,r,t} \geq Y_{i,r,t}, \\ Y_{i,r,t} - SL_{i,r,t} & \text{else.} \end{cases} \tag{2}$$

Following the production planning heuristic sketched above, the replenishment level $Y_{i,r,t}$ is set such that the estimated firm's demand is covered with parbability χ, where χ is denoted as the service level of the firm. The firm assumes that demand follows a normal distribution and hence the replenishment level can be expressed as

$$Y_{i,r,t} = \underbrace{\hat{D}_{i,r,t}}_{\text{Expected demand}} + \underbrace{q_\chi \cdot \sqrt{\hat{\sigma}^2_{\hat{D}i,r,t}}}_{\text{Buffer}}, \tag{3}$$

where $\hat{D}_{i,r,t}(p^*)$ is the expected demand, $\hat{\sigma}^2_{\hat{D}i,r,t}$ the estimated variance of the demand distribution and q_χ the χ-quantile of the standard Gaussianl distribution with mean 0 and variance 1.

The sum of the planned delivery volumes for all malls yields the desired output

$$\tilde{Q}_{i,t} = \sum_{r=1}^{R} \tilde{D}_{i,r,t} \tag{4}$$

which is the planned production quantity entering the input factor calculation and the financial planning in t.

3.2.2 Factor demand

Consumption goods producers need physical capital and labor for production. A firm i has a capital stock $K_{i,t}$ that is composed of different vintages of the production technology v with $v = 1, ..., V$,

$$K_{i,t} = \sum_{v=1}^{V} K^v_{i,t}. \tag{5}$$

The accumulation of physical capital by a consumption goods producer follows

$$K_{i,t+1} = \sum_{v=1}^{V} (1-\delta) K^v_{i,t} + \sum_{v=1}^{V} I^v_{i,t} \tag{6}$$

where δ is the depreciation rate and $I^v_{i,t} \geq 0$ is the gross investment in vintage v.

The production technology in the consumption goods sector is represented by a Leontief type production function with complementarities between the qualities of the different vintages of the investment good and the specific skill level of employees for using these types of technologies. Vintages are deployed for production in descending order by using the best vintage first. For each vintage the effective productivity is determined by the minimum of its productivity and the average level of relevant specific skills of the workers. Accordingly, output for a consumption goods producer is given by

$$Q_{i,t} = \sum_{v=1}^{V} \min \left[K^v_{i,t}, \max \left[0, L_{i,t} - \sum_{k=v+1}^{V} K^k_{i,t} \right] \right] \cdot \min \left[A^v, B_{i,t} \right], \tag{7}$$

where A^v is the productivity of vintage v and $B_{i,t}$ denotes the average specific skill level in firms. Complementarity between the quality of physical capital and worker skills is an empirically well established fact. The fact that the considered production function takes into account the vintage structure of the captial stock and also allows firms to select among different available vintages enables us to capture the effect of workers' skills on the incentives of firms to invest new technologies. Empirical evidence that firms' technology choices are indeed influenced by the skill level of their work force can for example be found in Piva and Vivarelli (2009).

An important parameter for the input factor determination is the potential output volume that can technically be produced with the present capital stock of the firm. This feasible output level $\hat{Q}_{i,t}$ is computed according to

$$\hat{Q}_{i,t} = \sum_{v=1}^{V} (1-\delta) K^v_{i,t} \cdot \min \left[A^v, B_{i,t} \right]. \tag{8}$$

With respect to $\hat{Q}_{i,t}$ and the desired output level $\tilde{Q}_{i,t}$ we have to consider two cases with different implications for the capital demand to be purchased at the capital goods market:

1. If $\hat{Q}_{i,t} \geq \tilde{Q}_{i,t}$: In that case the desired output can be produced with the current capital stock and no additional investments are necessary. We have $I_{i,t} = 0$ and the labor input is computed by taking the labor productivity of the last month into account:

$$\tilde{L}_{i,t} = \tilde{Q}_{i,t} \cdot \frac{L_{i,t-1}}{Q_{i,t-1}}. \tag{9}$$

2. If $\hat{Q}_{i,t} < \tilde{Q}_{i,t}$: Here we have positive investments $I_{i,t} > 0$; the amount depends on the outcome of the vintage choice. If v is the selected vintage, the investment volume is

$$I_{i,t} = \frac{\tilde{Q}_{i,t} - \hat{Q}_{i,t}}{\min\left[A^v, B_{i,t}\right]} \tag{10}$$

and the labor demand becomes

$$\tilde{L}_{i,t} = K_{i,t-1}(1-\delta) + I_{i,t}. \tag{11}$$

Depending on whether the labor demand exceeds or deceeds the current workforce the firm has to either hire additional workers or dismiss redundant workers.

Vintage choice The consumption goods firm chooses from a set of vintages $v = 1, ..., V$ which differ regarding their productivity A^v. The decision in which vintage to invest, depends on a comparison of the effective productivities and the corresponding prices. For this decision the complementarity between specific skills and technology, $\min[A^v, B_{i,t}]$, plays an important role: due to the inertia of the specific skill adaptation, the advantage of a better vintage with $A^v > B_{i,t}$ cannot be fully taken into account immediately, as the productivity gap is only closing over time. Therefore, the firm computes a discounted sum of estimated effective productivities over a fixed time horizon S. The specific skill evolution is estimated for each time step within this period $[t, t+S]$ along the same formula that is used by households for their individual skill adaptation (see Equation 92 in Section 3.6.5) , with the firm's mean general skill level $B^{gen}_{i,t}$ and mean specific skill level $B_{i,t}$. Formally, we have

$$\hat{A}^{eff}_{i,t}(v) = \sum_{s=0}^{S} \left(\frac{1}{1+\rho}\right)^s \min[A^v, \hat{B}_{i,t+s}(A^v)], \tag{12}$$

where ρ is the discount rate. The estimated adaptation of specific skills follows

$$\hat{B}_{i,t+s} = \hat{B}_{i,t+s-1} + \chi(B^{gen}_{i,t}) \cdot \max\left[A^v - \hat{B}_{i,t+s-1}, 0\right]. \tag{13}$$

The vintage choice follows a logit model. The decision is random where the probabilities depend on the ratios of effective productivity and prices $\frac{\check{A}^{Eff}_{i,t}(v)}{p^v_t}$. The higher the ratio is for a certain vintage the higher is the probability to buy it. Formally, we have for vintage v:

$$Prob_{i,v,t} = \frac{\exp(\gamma^v \log(\frac{\check{A}^{Eff}_{i,t}(v)}{p^v_t}))}{\sum_{v=1}^{V} \exp(\gamma^v \log(\frac{\check{A}^{Eff}_{i,t}(v)}{p^v_t}))}. \tag{14}$$

In several parts of the the Eurace@Unibi model choices of decision makers are described by logit models. These models are well suited to capture decisions where individuals try to maximize some objective function which depends on observable and unobservable variables. A more extensive discussion of the foundations of logit models in the context of consumer choices is given in Section 3.6.4.

3.2.3 Labor demand

Each worker h has two dimensions of human capital endowments namely an observable exogenously given general skill level b_h^{gen} and an unobservable endogenously changing specific skill level $b_{h,t}$. General skills can be interpreted as formal qualification while specific skills are abilities obtained on-the-job reflecting the productivity of each worker (see Section 3.6.5 for more details).

If the firms plan to extend the production (i.e. their labor demand is higher than the current workforce, $\tilde{L}_{i,t} > L_{i,t}$) they post vacancies and corresponding wage offers. The wage offer has two constituent parts. The first part is the market driven base wage $w_{i,t}^{base}$. The base wage is paid per unit of specific skill. If the firm can not fill its vacancies and the number of unfilled vacancies exceed some threshold $\overline{v} > 0$ the firm raises the base wage offer by a fraction φ to attract more workers such that

$$w_{i,t+1}^{base} = (1+\varphi) w_{i,t}^{base}. \tag{15}$$

The second part is related to the specific skills. Since the specific skills represent the (maximal) productivity of the employees the wage $w_{i,t}$ is higher for higher specific skills. For each of the general-skill groups the firm i offers different wages $w_{i,t,g}^{O}$ in period t. The wage offers are given by

$$w_{i,t,g}^{O} = w_{i,t}^{base} \times \bar{B}_{i,t,g} \tag{16}$$

where $\bar{B}_{i,t,g}$ are the average specific skills of all employees with general skill g in the firm. The underlying assumption of this determination of wage offers is that firms can observe general but not specific skills of job applicants. Therefore they use the average specific skills of all employees with general skill g in the firm in order to estimate the specific skills of an applicant with general skill level g. If the number of applicants is higher than the number of vacancies the firms choose between all applicants with respect to their general skill levels. An applicant with high general skills is more likely to receive a job offer than an applicant with low general skill. The probability that firm i chooses applicant a is determined using a multi-nominal logit model.

$$Prob_{i,h,t} = \frac{\exp(\gamma^{gen} b_a^{gen})}{\sum_{a \in A} \exp(\gamma^{gen} b_a^{gen})}. \tag{17}$$

The parameter γ^{gen} steers the influence of the general skill level of an applicant on his probability of being hired. The firm sends as many job offers as it has vacancies to fill. If the number of applicants is lower than the number of vacancies the firm sends job offers to all of the applicants. In case of downsizing the incumbent workforce $\tilde{L}_{i,t} < L_{i,t}$, the firm dismisses workers with lowest general skill levels first, because they have generally a lower speed of learning. In order to represent factors which lead to dismissals other than a decreased labor demand (i.e. on-the-job-search, workers quitting) the firm dismisses a fraction of employees in each month ϱ which is randomly drawn from the equally distributed interval $[\varrho^{low}, \varrho^{up}]$.

At the end of each month the wages of employees are increased proportional to the (regional) productivity growth. This can be interpreted as a simplified collective wage bargaining.

3.2.4 Financial management

At the end of the month (the firm specific production cycle) the firm computes its income statement to determine its profits. The firm balance sheet and cash flow are shown in Table 2 and 3.

The variable costs of the firm consists of the actual labour costs and a proportion of the total investments that are accounted for in this month. The amortization period of fixed capital investments equals the number of installment periods of a loan.

$$C_{i,t}^{Var} = w_{i,t}L_{i,t} + \frac{p_t^v \cdot I_t^v}{T^L}. \tag{18}$$

Here $v \in V$ stands for the vintage of capital and T^L is the loan period.

The fixed costs consisting of the irreversible capital expenditures of the previous periods $(t - T^L, ..., t-1)$ that are accounted for in t, plus interest on all outstanding loans over those periods, is given by

$$C_{i,t}^{Fix} = \sum_{l=1}^{T^L} \frac{p_{t-l}^v \cdot I_{t-l}^v}{T^L} + \sum_{l=1}^{T^L} r_{i,t-l} \cdot Loan_{i,t-l}. \tag{19}$$

The monthly realized profit of a consumption goods producer equals the sales revenues plus interest received on firm deposits minus production costs (i.e. fixed and variable costs) in the period that just ended.

$$\pi_{i,t} = R_{i,t} + r^b M_{i,t} - (C_{i,t}^{Fix} + C_{i,t}^{Var}) \tag{20}$$

$$= R_{i,t} + r^b M_{i,t} - \sum_{l=1}^{T^L} \frac{p_{t-l}^v \cdot I_{t-l}^v}{T^L} - \sum_{l=1}^{T^L} r_{i,t-l} \cdot Loan_{i,t-l} - \frac{p_t^v \cdot I_t^v}{T^L} - w_{i,t}L_{i,t}. \tag{21}$$

In case of positive profits, the firm pays taxes and dividends.[2] The net earnings (or losses) are

$$\pi_{i,t}^{net} = \min\left[\pi_{i,t}, (1-\tau)\pi_{i,t}\right] \tag{23}$$

$$Div_{i,t} = \max\left[0, d\pi_{i,t}^{net}\right]. \tag{24}$$

The retained net earnings (or losses) after interests, taxes and dividends are added to a payment account $M_{i,t}$

$$\pi_{i,t}^{ret} = \min\left[\pi_{i,t}, (1-d)(1-\tau)\pi_{i,t}\right] \tag{25}$$

$$M_{i,t+1} = M_{i,t} + \pi_{i,t}^{ret} \tag{26}$$

The actual total expenditures in period t are:

$$Exp_{i,t} = \underbrace{\tau \max\left[0, \pi_{i,t}\right]}_{\text{Taxes}} + \underbrace{d(1-\tau)\max\left[0, \pi_{i,t}\right]}_{\text{Dividends}} \tag{27}$$

$$+ \underbrace{p_{i,t}^v \cdot I_{i,t}^v}_{\text{Capital bill}} + \underbrace{w_{i,t}L_{i,t}}_{\text{Wage bill}} + \underbrace{\sum_{l=1}^{T^L} \frac{Loan_{i,t-l}}{T^L}}_{\text{Debt installment}} + \underbrace{\sum_{l=1}^{T^L} r_{i,t-l} \cdot Loan_{i,t-l}}_{\text{Interests}} \tag{28}$$

[2] A slightly altered rule for the dividend rate d is being used in case the firm's payment account exceeds a threshold level $\bar{d}$ that depends on the average revenues over the last four months: $\bar{d} = 0.5 \sum_{\tau=1}^{4} R_{t-\tau}^f$.

$$d = \begin{cases} d^{fix} & \text{if } M_t^f \leq \bar{d} \\ 1 & \text{otherwise.} \end{cases} \tag{22}$$

The rule states that if the payment account of a firm is below the threshold level $\bar{d}$ then it pays out the default fixed dividend rate $d^{fix} = 0.70$. If the payment account is above the threshold level $\bar{d}$ the firm pays out 100% of net earnings in dividends. This rule was instaled to prevent firms from hoarding money on the payment account, which resulted to be detrimental for economic performance as it works disadvantageous to the demand side.

Note that the expenditures in Eqn.(28) differ from the costs taken into account in the profits in Eqn.(21). In the expenditures the full capital bill is taken into account, whereas in the costs we only include calculatory capital costs that are an amortized proportion of the current investments in this period.

3.2.5 Firm credit demand

At the start of period $t+1$, the total liquidity needs to finance the next production cycle consist of the planned production costs, i.e. the wage bill and investments. The firm also has financial commitments carried over from the previous period, such as taxes and dividends on profits, debt principle and interest payments.

The actual liquidity needs that should be financed at the start of period $t+1$ are as follows

$$Liq_{i,t+1} = \underbrace{\tau \max[0, \pi_{i,t}]}_{\text{Taxes}} + \underbrace{d(1-\tau)\max[0, \pi_{i,t}]}_{\text{Dividends}} \tag{29}$$

$$+ \underbrace{p^v_{i,t+1} \cdot I^v_{i,t+1}}_{\text{Capital bill}} + \underbrace{w_{i,t+1} L_{i,t+1}}_{\text{Wage bill}} + \underbrace{\sum_{l=0}^{T^L-1} \frac{Loan_{i,t-l}}{T^L}}_{\text{Debt principle}} + \underbrace{\sum_{l=0}^{T^L-1} r_{i,t-l} \cdot Loan_{i,t-l}}_{\text{Interests}} \tag{30}$$

Note that this equation contains some unknowns at the time that liquidity needs are determined: the actual price of capital and the wages are unknown. The firm assumes $p^v_{i,t+1} = p^v_{i,t}$ and $w_{i,t+1} = \bar{w}_{i,t}$, the average wage of the firm.

This implies that the actual expenditures in (28) may differ from the liquidity needs determined by (30). If so, the firm may have a liquidity crisis if it turns out that capital or wages are more expensive than expected. In such a case the firm rescales production to fit its available liquidity.

The payment account is used as primary source to cover all expenses. If a firm does not have sufficient internal financial resources, it first depletes the payment account before resorting to external financing on the credit market. This follows the Pecking Order theory (Liesz, 2005) in which firms finance their expenditures according to a hierarchy of risk, using the least risky form of financing before more risky forms.

The demand for bank loans is the remaining part of the total liquidity needs that cannot be financed internally from the payment account.

$$\tilde{Loan}_{i,t} = \max\{0, Liq_{i,t+1} - M_{i,t+1}\} \tag{31}$$

3.2.6 Production

In the previous stages of the production process, the consumption goods producer has determined a desired production quantity as well as the input factor requirements and financial resources that are necessary to produce the desired output. If the internal financial resources are not sufficient to cover all expenses related to production plus the financial commitments (i.e. taxes, interests and debt repayments, and dividends), then the firm enters the credit market in order to raise external financing.

The financial commitments are preferential payments which have to be serviced before the production expenditures. Therefore, once the financial management has finished and the financial commitments have been paid, the firm should reevaluate the financial resources available

Table 2: Firm balance sheet.

Assets	**Liabilities**
M_i: liquidity deposited at a given *bank* $+p_iR_i$ $-w_iL_i - p^vI_i^v - T_i$ $+\sum_b \Delta Debt_{i,b}$ $+r^bM_i - \sum_b r^bLoan_{i,b}$ $-d_iN_i$	$Debt_{i,b}$: debts to *banks* $+Loan_{i,b}$ $-\sum_b \Delta D_{i,b}$
Inv_i: value of local inventories at *malls* $-p_iR_i$ $+p_iQ_i$	E_i: equity
K_i: value of physical capital $+I_i$	

Table 3: Firm cash flow.

Ingoing	**Outgoing**
p_iR_i Sales revenues p_iQ_i Output r^bM_i Interest on deposits $\sum_b Loan_{i,b}$ New credit from banks	w_iL_i Labour costs $p^vI_i^v$ Investment costs T_i Tax payment $\sum_b \Delta Debt_{i,b}$ Debt installment payments $\sum_b r^bLoan_{i,b}$ Interest payment d_iN_i Dividend payment
Total income	Total expenses

to cover all planned production expenses. If the available resources are not sufficient to cover the planned output, then the firm has to downscale the planned output level such that the production costs can be financed with the residual funds.

Thereafter, the firm enters the factor markets in the order of first the labor market and then the capital goods market. Firm's activities at the labor market depend on the labor demand and the current size of the labor force (see Section 3.2.3). If the labor demand exceeds the current number of employees the firm has to hire more workers. On the other hand, if the firm needs less workers than its current workforce the firm dismisses workers.

The structure of the labor market allows for frictions so that even if there is a persistent unemployment the labor demand is not necessarily satisfied completely. When a firm is not able to fill all its vacancies it has to adapt the production quantity to the level that can be produced with the labor force. In the current model implementation we assume that the capital good is offered at infinitely elastic supply hence the firm is never rationed at the capital goods market.

The actual production quantity is then determined according to the Leontief production

function

$$Q_{i,t} = \sum_{v=1}^{V} \min \left[K_{i,t}^{v}, \max \left[0, L_{i,t} - \sum_{k=v+1}^{V} K_{i,t}^{k} \right] \right] \cdot \min \left[A^{v}, B_{i,t} \right]. \tag{32}$$

The output of the production process is distributed among the malls in proportion to the planned delivery volumes $\tilde{D}_{i,r,t}$. The delivery volume to mall r is

$$D_{i,r,t} = \frac{\tilde{Q}_{i,t}}{Q_{i,t}} \cdot \tilde{D}_{i,r,t}. \tag{33}$$

3.2.7 Pricing decision

Consumption goods producers set the price of their products once a year. This relatively long period between two price changes of an individual firm is consistent with empirical observations for US firms (Blinder (1991)) and for firms located in the Euro area (Fabiani et al. (2006)). These papers are based on surveys concerning firms' frequency of price changes and both studies find that the median firm changes its price once a year.

The pricing rule presented here is inspired by the price setting described in (Nagle and Hogan, 2006, ch.7), a standard volume on strategic pricing. They describe the price setting as a three-stage process with a preliminary segment pricing stage, an optimization stage and the final implementation stage. Due to the highly stylized modeling of the consumption goods firms in our model, we can abstract from the first and the last stage and let the pricing decision depend only on the optimization stage. Firms seek for a profit-maximizing price taking into account the trade-off between price, sales and costs. However, due to the stochastic and complex nature of the model and the arising uncertainty of future developments this optimization procedure can only maximize the estimated profit of the firm.

In order to estimate possible impacts of price changes on the sales and the costs and finally on its profitability, the firm has to collect and process manifold information. Therefore, the firm passes a sequence of activities:

1. A market analysis for gathering information that allows estimating how demand responds to price changes.
2. An output analysis for drafting a preliminary production plan for a range of prices. These plans take into account the demand estimates as well as the inventory policy of the firm.
3. A cost analysis for each of the preliminary output plans including an estimate of the factor prices.
4. A profit analysis which leads to the final pricing decision.

Market analysis An important aspect for the profit estimate of different candidate prices is a suitable estimation of the demand function. To obtain an estimated demand curve, the consumption goods producers carry out market research. We assume there are two steps required: the first step is to estimate the overall development of the market. This is to know the potential market size for which the firm competes with its competitors. The second step is a estimation of the market shares that the firm can achieve given the estimated price sensitivity of consumers and their expectations about the prices of the competitors.

For the determination of the expected market trend we assume backward looking expectations. Let t be the month when the firm undertakes the market research and $S_{t-\iota}^{Tot}$ the real

market size (i.e. total sales in the market) in month $t-\iota$. For determining a global market trend the firm applies a linear regression model of observed market sizes $S_{t-\iota}^{Tot}$ for a fixed number of previous months. The expected market size in the future month $t+\tau$, $\hat{S}_{t+\tau}^{Tot}$ is then determined by

$$\hat{S}_{t+\tau}^{Tot} = \hat{S}_{t}^{Tot} + \hat{b}^{S} \cdot \tau, \tag{34}$$

where τ counts the months starting from the current month t and $\hat{b}^S$ the slope of the linear regression.

In the second step firm specific market potentials are evaluated. It is determined how a firm's position relative to its competitors changes as the price is changed. To this end, the firm carries out consumer surveys. The procedures builds on what is known as simulated purchase surveys in the relevant literature (e.g. Nagle and Hogan, 2006, pp. 300). Such surveys are performed by presenting consumers a sample of products and prices of the firm under consideration and of its competitor. The consumers are asked which product they would choose. Based on the results of such surveys the sensitivity of buyers with respect to price changes is estimated.

The firm draws a representative sample of households. All participating households are asked to compare a firm's product with the set of the currently available rival products. The price of the firm's product is varied within a discrete price range; and for each price p^{MR} in this set, the households respond whether or not to buy the good at the specified price. This decision is a simulation of households' real purchasing decision subject to the same influencing factors as the regular weekly consumers choice: it is modeled as a random process with a probability to decide to buy the product of firm i at price p^{MR}, with given expectations for the prices of the competitors, determined by

$$prob = \frac{\exp(-\gamma^C \cdot \log p^{MR})}{\exp(-\gamma^C \cdot \log p^{MR}) + \sum_{\forall k \neq i} \exp(-\gamma^C \cdot \log \hat{p}_k))}, \tag{35}$$

where γ^C is a parameter for the consumer's sensitivity with respect to price differences of the available goods. $\hat{p}_k$ are the expectations of the prices of the competing products.

The firm collects the yes-no answers of the participants of the survey; dividing the number of positive responses at a price p^{MR} by the total sample size n^{Sample} yields a first estimate of the achievable market shares. Once the firm has gathered the relative frequencies, it runs a regression to find a relation between prices and achievable market shares given the expected prices of the competitors. The actual function of expected market shares is approximated by the exponential function

$$\hat{s} = \hat{b}^D \cdot \exp(-\hat{a}^D \cdot p^{MR}). \tag{36}$$

It should be pointed out that firms are not aware of the actual purchasing probabilities (35). The exponential choice in (36) is based on the observation that the market data can be explained much better by this form compared to a standard linear model.

Firms act in a dynamic environment with changing prices, but the market share estimation is a point estimation that does not take into account these dynamics. Consequently, the estimation can loose its goodness of fit when the actual characteristics of the competitors significantly deviate from the assumed competitive environment. This can cause problems particularly with regard to long planing periods. Particularly, if the price level increase due to a persistent inflation and the market share estimation is used for the whole planning period (e.g. the planning period for the pricing is one year). In order to alleviate this problem, a firm always carries out two surveys that differ only with respect to the expectations concerning competitors' prices. The first includes the current market situation while in the second the willingness to buy is queried

for the expected market environment at the end of the planning phase. The estimated market shares for a price p^{MR} for the months in between are determined by a linear combination. For a planning period of length T starting at t we have for month $t+\tau$ an estimated market share of

$$\hat{s}_{t+\tau} = \frac{(T-1)-\tau}{T-1} \cdot \hat{s}_t + \frac{\tau}{T-1} \cdot \hat{s}_{t+T-1}. \tag{37}$$

Based on the results of the market analysis the firm can construct the estimated demand function. The firm combines the estimated trend of the total market size $\hat{S}^{Tot}_{t+\tau}$ with the expected market share $\hat{s}_{t+\tau,\tilde{q}}$. The expected demand curve, estimated for the τth month after t writes

$$\hat{D}_{i,t+\tau}(p^{MR}) = \hat{S}^{Tot}_{t+\tau} \cdot \hat{s}_{t+\tau}(p^{MR}). \tag{38}$$

Preliminary output planing In order to find a profit maximizing price the firm does not only need an estimate on the sales but also an estimate of the production costs. Therefore, the firm requires information about the production quantities during the planning period. On that account the consumption goods firm has to set up a preliminary production plan for the following months under consideration. The drawing up of this production plan is similar to the actual monthly production planing as described in Section 3.2.1.

The firm derives its preliminary production plan from the estimated demand function and a buffer stock. The buffer allows to serve unforeseeable demand realizations above average. The size of the buffer depends on the level of service, i.e. the percentage of the theoretically possible demand that the firm still wants to satisfy.

In contrast to the actual monthly output planning, in the preliminary output planning for the price setting the firms abstract from the local mall inventories and consider only global inventories. At the beginning of each production cycle, the firms plan to refill their inventories up to a level that corresponds to the service level of their demand. Let χ be the level of service and $\hat{\sigma}^2_{\hat{D}i,t+\tau}$ the estimated variance of the demand function in $t+\tau$, then the critical inventory stock is determined by

$$\hat{Y}_{i,t+\tau} = \underbrace{\hat{D}_{i,t+\tau}}_{\text{Expected demand}} + \underbrace{q_\chi \cdot \sqrt{\hat{\sigma}^2_{\hat{D}i,t+\tau}}}_{\text{Buffer}}, \tag{39}$$

where q_χ is the χ- quantile of the normal distribution.

The expected production quantity for period $t+\tau$ is then simply the difference of the critical inventory stock $\hat{Y}_{i,t+\tau}$ and the expected current stock. The expected current stock in $t+\tau$ can be derived from the replenished inventory of the previous period minus the expected demand in $t+\tau-1$, thus we have

$$\hat{Q}_{i,t+\tau} = \hat{Y}_{i,t+\tau} - \underbrace{\hat{Y}_{i,t+\tau-1} - \hat{D}_{i,t+\tau-1}}_{\text{Current stock at the beginning of } t+\tau}. \tag{40}$$

The starting value of the inventory stock in the first month of the planning period t is the total buffer that has been used in the determination of the actual production quantity in the previous period $t-1$.

Cost analysis Production costs are driven by the usage of input factors in the production process. These costs are essentially wage costs and costs arising from the use of capital. For the estimate of costs it is important to estimate on the one hand the requirements for input factors and external financing over time, and on the other hand the development of factor prices.

The firm computes the required input factor demand for labor and capital for each period $t+\tau$ with $\tau \in \{0, ..., 11\}$. The first step to determine the expected requirements of the input factors labor and capital in $t+\tau$ is the computation of the maximum output that the firm can produce with the available capital stock at the beginning of $t+\tau$. The feasible output, derived from the Leontief production function, is the sum of the depreciated effective capital amounts of all vintages held in the former period,

$$\tilde{Q}_{i,t+\tau} = \sum_{v=1}^{V} (1-\rho)\tilde{K}^{v}_{i,t+\tau-1} \cdot \min[A^{v}, \hat{B}_{i,t+\tau}], \tag{41}$$

where $\hat{B}_{i,t+\tau}$ denotes the estimated mean specific skill level of firm's workforce and A^v the technical productivity of vintage v. The evolution of the prospective capital stock from period t, when the firm carries out the analysis, to the planning month before the currently considered period, $t+\tau-1$, is driven by depreciations and investments, and the depreciation of intermediate investments. Formally, the evolution is represented by

$$\tilde{K}^{v}_{i,t+\tau-1} = \begin{cases} (1-\rho)^{\tau-2} K^{v}_{i,t-1} + \sum_{l=0}^{\tau-1} (1-\rho)^{\tau-1-l} \cdot \hat{I}^{v}_{i,t+l} & \text{if } v = v\star, \\ (1-\rho)^{\tau-2} K^{v}_{i,t-1} & \text{else.} \end{cases} \tag{42}$$

As there are several vintages $v = 1, ..., V$ of the production technology, the firm has to decide in which vintage to invest. Here it is assumed that the vintage $v\star$ selected in the periodic vintage choice for production in t is also used for the cost planning for all months under consideration.

The mean specific skills $\hat{B}_{i,t+\tau}$ in equation (41) are estimated by applying the individual specific skill adaptation formula of workers with the mean of specific skills and the productivity of the deployed capital stock in the previous period $\hat{A}_{i,t+\tau-1}$, as well as the average of general skills of a firm's employees as parameters (see Section 3.6.5). Put formally, we have

$$\hat{B}_{i,t+\tau} = \hat{B}_{i,t+\tau-1} + \chi(B^{gen}_{i,t}) \cdot \max[\hat{A}_{i,t+\tau-1} - \hat{B}_{i,t+\tau-1}, 0]. \tag{43}$$

Depending on the level of feasible output on the one hand and the intended production quantity on the other, the firm has to consider two different cases:

1. The feasible output $\tilde{Q}_{i,t+\tau}$ is less than or equal to the expected output $\hat{Q}_{i,t+\tau}$: In that case the capital stock of the firm is not sufficient for producing the desired output and additional investments are necessary. The needed capital investments are determined by

$$\hat{I}^{v\star}_{i,t+\tau} = \frac{\hat{Q}_{i,t+\tau} - \tilde{Q}_{i,t+\tau}}{\min[A^{v\star}, \hat{B}_{i,t+\tau}]}, \tag{44}$$

and the resulting labor demand is

$$\hat{L}_{i,t+\tau} = \sum_{v=1}^{V} (1-\rho)\tilde{K}^{v}_{i,t+\tau-1} + \hat{I}^{v\star}_{i,t+\tau}. \tag{45}$$

2. In the second case the feasible output is greater than the desired one, $\tilde{Q}_{i,t+\tau} > \hat{Q}_{i,t+\tau}$: Here only a part of firm's capital stock is deployed. It is assumed that the firm starts using the best vintages of its production technology first. Because of the Leontieff production function the labour demand can be directly derived from the capital requirements. Labour demand is computed according to

$$\hat{L}_{i,t+\tau} = \sum_{v=1}^{V} \min \left[\tilde{K}^{v}_{i,t+\tau}, \max \left[0, \hat{Q}_{i,t+\tau} - \sum_{k=v+1}^{V} (\tilde{K}^{k}_{i,t+\tau} \cdot \min[A^k, \hat{B}_{i,t+\tau}]) \right] \right]. \tag{46}$$

As the capital stock is used with less than its capacity no additional investment are required, $\hat{I}^{v\star}_{i,t+\tau} = 0$.

The prospective variable costs are related to the estimated labor input and, if additional investments are required, the portion of capital expenses in $t+\tau$ that is apportioned to this month. The estimated variable costs are

$$\hat{C}^{Var}_{i,t+\tau} = \underbrace{\hat{w}_{i,t+\tau} \cdot \hat{L}_{i,t+\tau}}_{\text{Labour costs}} + \underbrace{\frac{\hat{I}^{v\star}_{i,t+\tau} \cdot p^{v\star}_{t}}{T^{Loan}}}_{\text{Variable capital costs}} . \tag{47}$$

$\hat{w}_{i,t+\tau}$ is the estimated mean wage in month $t+\tau$. It is determined by

$$\begin{aligned} \hat{w}_{i,t+\tau} = & \underbrace{\hat{w}_{i,t+\tau-1} \cdot (1+g^P) \cdot \min[1, \frac{\hat{L}_{i,t+\tau-1}}{\hat{L}_{i,t+\tau}}]}_{1} + \\ & \underbrace{w^{base}_{i,t} \min[\hat{A}_{i,t+\tau-1}, \hat{B}_{i,t+\tau-1}] \cdot \max[0, \frac{\hat{L}_{i,t+\tau} - \hat{L}_{i,t+\tau-1}}{\hat{L}_{i,t+\tau}}]}_{2} \end{aligned} \tag{48}$$

Equation (48) takes into account two different sources for increasing wages. The first term captures the wage increase that is due to the productivity growth in the economy. The wage of employed workers increases with the same monthly growth rate as the average productivity in a firm's home region. In order to estimate the wage increase of incumbent workers the firm determines the average regional productivity growth rate of the last 12 months, g^P. The second term captures the change in the mean wage induced by newly hired workers. If a worker accepts a job, the first wage corresponds to the wage offer of her general skill group. This wage offer is computed by multiplying a base wage offer with the effective productivity of the skill group (minimum of average specific skills and firm's technology). For the cost estimation, however, the firm abstracts from skill groups and considers only firm specific averages.

For estimating the fixed costs the firm has to sum the portions of irreversible capital expenditures spent in previous periods which have to be accounted for in $t+\tau$. Furthermore, the firm estimates the credit costs by summing the interest payments for all loans which are still open in $t+\tau$. We have

$$\hat{C}^{Fix}_{i,t+\tau} = \underbrace{\sum_{l=1}^{T^{Loan}} \frac{\hat{I}^{v\star}_{i,t+\tau-l} \cdot p^{v\star}_{\min[t,t+\tau-l]}}{T^{Loan}}}_{\text{Fixed capital costs}} + \underbrace{\sum_{l=1}^{T^{Loan}} \hat{Loan}_{i,t+\tau,t+\tau-l} \cdot r_{i,t+\tau-l}}_{\text{Credit costs}} . \tag{49}$$

Thereby, $v\star$ is the vintage selected for the investment in $t+\tau-l$ where for $l<\tau$ the selected vintage $v\star$ is $v\star$. $\hat{Loan}_{i,t+\tau,t+\tau-l}$ indicates the current value of a loan in the period $t+\tau$ (second subscript) which has been raised in $t+\tau-l$ (third subscript). $r_{i,t+\tau-l}$ is the interest rate for which the loan was accepted.

The expected demand for credit in $t+\tau$ is derived from a comparison of the prospective expenses and the financial resources that are available. This information stems from an estimate of the payment account which takes all intermediate inflows and outflows into account. At the beginning of $t+\tau$ the expected financial resources are given by $\hat{M}_{i,t+\tau-1}$. The firm has to pay a percentage τ of its previous profits $\hat{\pi}_{i,t+\tau-1}$ as taxes and a percentage d of the net profits as dividends. Then it has to pay the wage bill and investments for production in $t+\tau$, the debt installment, and interest payments. In sum, one has

$$\begin{aligned}\hat{Exp}_{i,t+\tau} = &\underbrace{tax\cdot\max[0,\hat{\pi}_{i,t+\tau-1}]}_{\text{Taxes}} + \underbrace{d\cdot(1-tax)\max[0,\hat{\pi}_{i,t+\tau-1}]}_{\text{Dividends}} + \underbrace{\hat{w}_{i,t+\tau}\cdot\hat{L}_{i,t+\tau}}_{\text{Wage bill}} \\ &+ \underbrace{\hat{I}^{v\star}_{i,t+\tau}\cdot p^{v\star}_{t}}_{\text{Capital bill}} + \underbrace{\sum_{l=1}^{T^{Loan}}\left(\hat{Loan}_{i,t+\tau,t+\tau-l}\cdot r_{i,t+\tau-l} + \frac{\hat{Loan}_{i,t+\tau-l,t+\tau-l}}{T^{Loan}}\right)}_{\text{Interest and debt installment payments}}.\end{aligned} \tag{50}$$

Additional credits are required if the expenses $\hat{Exp}_{i,t+\tau}$ are not completely covered by the available cash,

$$\hat{Loan}_{i,t+\tau,t+\tau} = \max[0, \hat{Exp}_{i,t+\tau} - \hat{M}_{i,t+\tau-1}]. \tag{51}$$

The firm adds the expected revenues $\hat{R}_{i,t+\tau}$ and the additionally raised credit on the notional payment account. At the end of the planning month the expected payment account balance is

$$\hat{M}_{i,t+\tau} = \hat{M}_{i,t+\tau-1} - \hat{Exp}_{i,t+\tau} + \hat{R}_{i,t+\tau} + \hat{Loan}_{i,t+\tau,t+\tau}. \tag{52}$$

Expected revenues generated by selling the good at a price p^{MR} in $t+\tau$ are computed by

$$\hat{R}_{i,t+\tau} = p^{MR}\cdot\hat{D}_{i,t+\tau}. \tag{53}$$

Profit analysis The firm evaluates expected profits for each of the candidate prices. As the price holds for one year, the firm compares the profitability of the alternatives with respect to the whole planning period. For month $t+\tau$ the estimated profit associated with the price p^{MR} is denoted by

$$\hat{\pi}_{i,t+\tau}(p^{MR}) = \hat{R}_{i,t+\tau}(p^{MR}) - (\hat{C}^{Var}_{i,t+\tau}(p^{MR}) + \hat{C}^{Fix}_{i,t+\tau}(p^{MR})). \tag{54}$$

The yearly profit is simply the sum of the monthly earnings. For the comparison of intertemporal payment flows, the firm considers the discounted sum:

$$\hat{\Pi}_t = \sum_{\tau=0}^{11}\frac{1}{(1+\delta)^{\tau}}\cdot\hat{\pi}_{i,t+\tau}. \tag{55}$$

The firm selects the price which yields the highest discounted profit $\hat{\Pi}_t$:

$$p_{i,t+\tau} = p^* = \arg\max_{p\in[\underline{p}^{MR}_{i,t},\bar{p}^{MR}_{i,t}]}\hat{\Pi}_t, \forall\tau = 0,...,11. \tag{56}$$

3.3 Banks

The bank has assets and liabilities (see table 4). The assets of a bank consist of cash reserves M^b and outstanding loans to firms L_i^b. Adding up households' deposits M_h^b, firms' deposits M_i^b, standing facilities D^b, and equity E^b gives liabilities.

From the standing facility with the Central Bank the banks can draw advances freely (D^b). They have to pay the base interest rate to the Central Bank ($-r^c D^b$). Banks pay dividends ($d^b N^b$) on outstanding equity shares. They do not purchase government bonds, and they do not purchase shares of other firms or banks.

Banks receive interest ($+r_i^b L_i^b$) and debt instalment payments (ΔL^b) on the outstanding loans to firms (L_i^b). All banks' cash reserves are deposited at the Central Bank overnight. The Central Bank pays the base interest on overnight deposits of the bank's liquid reserves ($r^c M^b$).

The deposit interest rate r^b that the bank pays on household and firm payment accounts is lower than the base rate r^c, while the interest on loans to firms r_i^b is higher than the base rate ($r^b < r^c < r_i^b$). This ensures that banks can make a profit.

The deposits interest rate r^b is determined as a mark-down on the base rate, while the interest on loans r_i^b depends on the firm-specific balance sheet, in particular on the probability of debt default.

The base rate of the Central Bank is constant as default. It is made endogeneous only in case of monetary policy experiments.

3.3.1 Bank accounting

Profits and dividends.
The profits consist of the margin between interests on loans and interests on deposits, plus (minus) any interest paid by (to) the Central Bank on overnight deposits (debt).

$$\pi_t^b = \sum_{i=0}^{F} \sum_{\tau=0}^{T^{loan}} r_{i,t-\tau}^b Loan_{i,t-\tau}^b - r^b(M_h^b + M_i^b) + r^c(M^b - D^b) \tag{57}$$

In case of positive profits, the bank pays taxes and dividends.

$$\pi_{b,t}^{net} = (1 - tax)\max\{0, \pi_t^b\} \tag{58}$$

$$T_t^b = tax \cdot \max\{0, \pi_t^b\} \tag{59}$$

$$Div_t^b = d(1 - tax)\max\{0, \pi_t^b\} \tag{60}$$

$$\pi_{b,t}^{ret} = [(1 - tax)(1 - d)] \cdot \max\{0, \pi_t^b\} \tag{61}$$

Cash flow.
Bank cash reserves fluctuate with deposits and withdrawals, interest payments, and finally also with taxes and dividends. The net profits (or losses) after taxes and dividends are added to the cash reserves M_t^b and deposited with the Central Bank.

$$M_{t+1}^b = M_t^b + \Delta M_h^b + \Delta M_i^b + (\pi_t^b - T_t^b - Div_t^b). \tag{62}$$

It is possible that due to the deposit mutations the bank's cash reserves become negative. We assume that the banks have a standing facility with the Central Bank from which the banks can draw advances freely. If bank liquidity is positive, the bank automatically pays off its Central Bank debt (ΔD^b) and receives interest on overnight deposits. If liquidity is negative, the bank automatically draws on Central Bank debt (ΔD^b), sets $M_{t+1}^b = 0$ and has to pay daily interest to the Central Bank.

3.3.2 Credit supply

The bank's ability to provide credit is restricted by a capital requirement and by a cash reserve ratio. For each individual loan request the bank decides on the credit conditions for the applying firm. The decision is on the amount of credit provided and the interest rate on the new loan. The bank records several characteristics of the applicant firm. Particularly, this is the firm total debt $Debt_{i,t}$, the firm's credit request $Loan_{i,t}$, and the firm equity $E_{i,t}$. These attributes enter into the loan conditions that the bank is offering to the firm.

The firm's probability of default depends on the debt/equity ratio of the firm after it would receive the new loan.

$$prob_i^{def} = 1 - e^{-(Debt_{i,t}+Loan_{i,t})/E_{i,t}}, \tag{63}$$

We assume that the expected loss given default (or Exposure at Default, EAD) is 100% of the value of a loan. This is the case if the collateral value of a loan is zero, i.e. debt is unsecured. Due to this assumption, the expected loss or credit risk (CR) of a loan is simply the probability of default times the value of the loan

$$cr_i^b = prob_i^{def} \cdot Loan_{i,t}. \tag{64}$$

The interest rate offered by bank b to firm i is an increasing function of the credit risk and reflects a risk premium that the bank charges.[3] The credit risk posed by firm i enters into the loan conditions as a markup over the Central Bank base interest rate.

The loan-specific interest rate thus depends on bank b's exposure to default of firm i:

$$r_i^b = r^c + \gamma_t^b cr_i^b \eta_{bt}. \tag{65}$$

Here γ_t^b is a behavioral parameter of the bank that determines the interest rate markup which is initialized to $\gamma_0^b = 0.5$, and $\eta_{bt} \sim U[0, 0.01]$ is a randomly uniform variable that represents the bank's operating costs.

The interest rate markup is determined in relation to the bank's profit growth rate:

$$g_{b,t} = (\pi_{b,t} - \pi_{b,t-1})/\pi_{b,t-1}$$

This assumption can be justified on the grounds that bank behavior is profit-driven. As long as profits are growing, the bank increases the interest rate markup to try to generate more profit. However, since firms are comparing the interest rates offered by multiple banks, this behavior will eventually lead the bank to price itself out of the market and profits decrease. Then the interest rate markup is decreased again.

The markup multiplier is extrapolated by the profit growth rate and a factor $\lambda(\gamma_t^b - \gamma_{t-1}^b)$, with a minimum markup level set to $\gamma = 0.02$.

$$\gamma_{t+1}^b = \gamma_t^b + \lambda(\gamma_t^b - \gamma_{t-1}^b) g_{b,t}. \tag{66}$$

[3] See Gaffeo et al. (2008) for a similar model of a credit market.

Minimal capital requirement If the bank grants the loan to firm i, the bank adds the associated credit risk to its total exposure. Each bank is required to maintain a capital adaquacy ratio, implying that banks have to observe a limited exposure to default risk. That is, bank equity must be greater or equal to a fraction $\kappa = 1/\alpha$ of the value of its risk-weighted assets (bank loans). This assumption is based on Basel II capital requirements (where κ equals $\kappa = 12\%$, i.e. $\alpha = 8.33$). Therefore, the bank's total exposure to credit risk is restricted by the multiplier α times the equity of the bank.

$$E^b \geq \kappa CR^b \Leftrightarrow CR^b \leq \alpha E^b \tag{67}$$

If this constraint is violated the bank stops providing new loans. Preexisting loans are still administered, firms continue to repay debt installments, and deposits of bank account holders continue to be serviced. From this we derive a "budget" of exposure units V^b that the bank can still provide to firms in the form of commercial loans:

$$V^b = \alpha E^b - CR^b. \tag{68}$$

The credit supply in the current period is restricted by the budget V^b. Firm i receives its full credit whenever the bank's total credit risk exposure remains below its limit. When the credit risk associated to the credit request $\tilde{Loan}_{i,t}$ superceeds the bank's limit, firm i receives only a proportion of its requested loan:

$$\bar{Loan}_{i,t} = \begin{cases} \tilde{Loan}_{i,t}, & CR^b + cr_i^b \leq \alpha E^b \\ (\alpha E^b - CR^b)/prob_i^{def}, & \text{else} \end{cases} \tag{69}$$

In terms of the excess V^b this reads:

$$\bar{Loan}_{i,t} = \begin{cases} \tilde{Loan}_{i,t}, & V^b \geq cr_i^b \\ V^b/prob_i^{def}, & V^b < cr_i^b \end{cases} \tag{70}$$

Liquidity constraint The banks must observe a minimum cash reserve requirement, that is, cash reserves must exceed a fraction β of deposits of households and firms

$$M^b \geq \beta(M_h^b + M_i^b) \tag{71}$$

From this the excess liquidity of the bank is derived as:

$$R^b = M^b - \beta(M_h^b + M_i^b). \tag{72}$$

The liquidity constraint (72) is added to the bank's constraint on total exposure in (67). If excess liquidity is sufficient to provide firm i its requested or constrained credit demand, then it is serviced in full. Otherwise it is credit rationed such that bank b attains its minimum cash reserve level. Hence, the credit supply is restricted to the excess liquidity:

$$Loan_{i,t} = \min\{\bar{Loan}_{i,t}, R^b\}. \tag{73}$$

Table 4: Bank balance sheet.

Assets	**Liabilities**
M^b: liquidity (cash reserves) (deposited at the *central bank*) $+\Delta M_h + \Delta M_i$ $-r^b(M_h^b + M_i^h)$ $+\sum \Delta Debt_i^b$ $-\sum \Delta Loan_i^b$ $+\sum r_i^b Loan_i^b$ $-r^c D^b$ $+r^c M^b$ $-Tax^b$ $-d^b N^b$ $+\Delta D^b(+)$ $-\Delta D^b(-)$ L_i^b: outstanding loans to firms $-\Delta Debt_i^b$ $+\Delta Loan_i^b$	M_h^b: households' deposits at the bank $+\Delta M_h$ M_i^b: firms' deposits at the bank $+\Delta M_i$ (withdrawals/deposits) D^b: standing facility (debts to the *central bank*) $+\Delta D^b(+)$ $-\Delta D^b(-)$ E^b: equity $-r^b(M_h^b + M_i^b)$ $+\sum r_i^b Loan_i^b$ $-r^c D^b$ $+r^c M^b$ $-Tax^b - d^b N^b$

Table 5: Bank cash flow.

Ingoing		**Outgoing**	
ΔM_h, ΔM_i	Deposits mutation		
$\sum \Delta Debt_i^b$	Firm loan installments	$\sum Loan_i^b$	New loans to firms
$\sum r_i^b Loan_i^b$	Firm interest payments		
$\Delta D^b(+)$	New ECB debt (standing facility)	$\Delta D^b(-)$	Reduction ECB debt
$r^c M^b$	Interest received from ECB	$r^c D^b$	Interest payment on ECB debt
$r^b(M_h^b + M_i^b)$	Interest on deposits	$d^b N^b$	Dividend payout
		Tax^b	Tax payment

3.4 Bankruptcy: Entry and Exit

Two types of bankruptcy are considered:

- illiquidity bankruptcy: After the credit market the firm was unsuccesful in raising all its external finances. It is unable to pay its financial commitments, i.e., taxes, debt instalments and interests. Firm equity is positive but it does not have enough liquidy to continue operations. It should raise enough funds to become liquid again.

- insolvency bankruptcy: The firm updates its balance sheet and checks for insolvency at the end of its production cycle when it has received the revenues from this month's sales. If the firm equity becomes negative it is insolvent and goes out of business. It has to perform a debt restructuring before it can continue operations.

Both types of bankruptcy may occur in the model. To describe what happens we define the following variables for each firm:

- $K_{i,t}$: nominal value of physical capital at market prices
- $MV_{i,t} = p_{i,t} SL_{i,t}$: market value of mall inventory (stock level)
- $M_{i,t}$: payment account
- $L^{tot}_{i,t}$: total loan
- $A^{tot}_{i,t} = K_{i,t} + MV_{i,t} + M_{i,t}$ (total assets)
- $E_{i,t} = A^{tot}_{i,t} - L^{tot}_{i,t}$ (equity)

3.4.1 Effects of bankruptcy

As soon as the firm declares bankruptcy, this has the following consequences

1. All current employees are fired and become unemployed. They receive unemployment benefits.
2. The firm suspends all production activities for a year. At the end of the period the firm reenters and production is restarted.
3. The capital stock remains in the firm, but is unproductive during the idle period.
4. Since we have modelled the financial market as an index, the shares of the firm continue to exist in the market index, but obviously pay no dividends. The households' portfolios of index shares are unaffected.
5. The local inventory stock at the malls is destroyed, representing the economic loss due to bankruptcy.
6. Debt renegotiation with the banks. For each loan, the firm defaults on a fraction of the loan. This is the bad debt that should be written off from the bank's balance sheet.

3.4.2 Debt renegotiation

To make it easier for re-entering firms to obtain new loans we should improve their debt-equity ratio and lower the risk of default. This makes it more likely for a bank to accept any future loan requests from such debt restructured firms. Debt renegotiation is addressed by re-scaling total loans L^{tot}, which is greater than total assets A^{tot} in the insolvency bankruptcy case, to a new value L^* lower than A^{tot}. The debt rescaling parameter ω is assumed constant across all firms

$$L^* = \omega A^{tot} \quad \text{with} \quad 0 \leq \omega \leq 1\,. \tag{74}$$

Accordingly, we let each bank b that is lending to the bankrupted firm be subject to a write-off w_b, i.e. a reduction of its assets (and consequently of its equity) proportional to the value of the loan L_{bj} granted to the bankrupted firm. Given that the total amount of write-off must be $L^{tot} - L^*$ and that $\sum_b L_{bj} = L^{tot}_j$, we have the bad debt for one loan of one bank to one firm

$$w_b = \frac{L^{tot_j} - L^*_j}{L^{tot}_j} L_{bj}\,. \tag{75}$$

Given the total firm assets A^{tot} after debt restructuring, the equity of the restructured firm is now positive,

$$E^* = (1-\omega)A^{tot} > 0\,. \tag{76}$$

The debt/equity-ratio after rescaling the debt is given by the constant: $L^*/E^* = \omega/(1-\omega)$.

For the illiquidity bankruptcy case the firm does not need to renegotiate its debt per se, since L^{tot} is already lower then A^{tot} beforehand. Since the firm is unable to pay its financial commitments it should raise new funds. It could do so either on the credit market or in the stock market by means of the issuing of new shares. However, since we have precluded firms from issuing new shares (for reasons of simplicity) we also allow illiquid firms to write down part of their debt.

Since the illiquid firm was previously unsuccessful to raise funds on the credit market, this was either due to a high debt/equity-ratio, or due to sheer unluck. Perhaps the firm was last in line and the bank did not have any excess reserves. Setting a low value for the debt/equity-ratio $L^*/E^* = \omega/(1-\omega)$ improves the firm's chances of being more successful in future loan requests. For this reason the value of the debt rescaling parameter ω is set to low values $\omega \leq 0.5$.

3.5 The Capital Goods Producer

In the current version of our model, the supply side of the capital goods market is still modeled in a simplified way meaning that there is a monopolistic capital goods firm that offers different vintages of the capital good at infinite supply elasticity. This firm is therefore able to satisfy all emerging capital demand required by consumption goods firms for production. Furthermore, the capital good is produced without input factor requirements and consequently without production costs and, in order to close the model, all revenues are channeled back into the economy by distributing them to households in the form of dividends. Despite these strong simplifications, the model includes the important aspects for a reasonable analysis of the interaction of the technological diffusion and the business cycle.

3.5.1 The Development of new Vintages

The capital good producer carries out research and development leading to new vintages of the capital good. The existing vintages $v = 1, .., V$ are therefore the result of previous innovations, and once a new vintage is launched, it is never taken off the market. The number of available vintages is hence increasing over time.

The innovation process is modeled as a random process but is not exogenously given. With a probability $prob^{Inno} > 0$ there is a successful innovation of a new vintage $V+1$. The productivity as the key characteristic of the technology is increased from the previous best practice by $\Delta q^{inv} > 0$, thus the new frontier technology features a productivity level of

$$A^{V+1} = (1 + \Delta q^{inv}) \cdot A^V\,. \tag{77}$$

The innovation probability $prob^{Inno}$ is determined as an increasing function of the R&D expenditures of the firm. In accordance with much of the empirical literature (see e.g. Cohen and Klepper (1996), Crepon and Mairessec (1998)) a size independent innovation intensity of firms is assumed, which means that R&D effort is proportional to the revenues of the capital goods firm. Hence, $prob^{Inno}$ is implemented as an incressing function of the mean revenues over the previous 6 months and consequently, the launch of a new vintage is more likely when the capital goods firm has a higher business volume.

3.5.2 The Pricing of Vintages

The pricing of the vintages is modeled as a combination of cost-based and value based pricing. Note that the costs for producing the capital good are not explicitly modeled in the present implementation, but we assume a proxy for the costs which have to be incorporated in the price setting. These as-if costs grow, similar to the evolution of the labor costs, with the same rate as the average productivity in the economy B_t^{Eco}. The cost-based price component has the same level for all vintages v and is denoted by

$$p_t^{Cost} = \frac{B_t^{Eco} - B_{t-1}^{Eco}}{B_{t-1}^{Eco}} \cdot p_{t-1}^{Cost}. \tag{78}$$

For the value-based price component the capital goods producer estimates the value that each vintage has for a reference firm whose workforce consists of the economy wide average levels for the specific skills (B_t^{Eco}) as well as general skills (b_{gen}^{Eco}). For the benchmark firm the capital goods producer computes the discounted sums of the expected effective productivities for each vintage v. This is done in an analogous manner as the estimate of the effective productivities by the consumption goods producers for their vintage choice, i.e. the capital goods firm estimates the evolution of specific skills $\hat{B}_{t+s}^{Eco}$ with respect to the average general skill level b_{gen}^{Eco}. For vintage v the discounted sum of effective productivities of the benchmark firm is

$$\bar{A}_t^v = \sum_{s=0}^{S} \left(\frac{1}{1+\rho}\right)^s \min[A^v, \hat{B}_{t+s}^{Eco}]. \tag{79}$$

$\bar{A}_t^v$ denotes the estimated future productivity of vintage v for a average firm in the economy. The relative utility of v compared to the lowest productivity vintage $v = 1$ gives then the ratio between their values. By assuming that the price for vintage $v = 1$ quoted in the previous period corresponds to its value, the value of v entering in the pricing can be estimated by

$$p_{v,t}^{value} = p_{t-1}^1 \cdot \frac{\bar{A}_t^v}{\bar{A}_t^1}. \tag{80}$$

The price of vintage v is finally set according to the linear combination

$$p_t^v = (1-\lambda) \cdot p_t^{cost} + \lambda \cdot p_{v,t}^{value}. \tag{81}$$

This rule can be interpreted as a proxy for a bargaining process, where the monopolistic capital goods producer on the one hand and the consumption goods firms on the other hand negotiate about the prices for the capital goods. The weighting factor λ ($0 \leq \lambda \leq 1$) is a parameter indicating the bargaining power of the capital goods firm; a high λ denotes that the capital goods firm can skim off the whole value of the vintage, whereas λ close to zero denotes that the capital firm has no power and can only set the price at its costs.

3.6 Household

3.6.1 Income determination

There are four different income types that a household can possibly receive:

1. The most important income is the labor income i.e. the monthly wage that a household receives if she is employed. The wage depends on the wage offer that the future employer offers when a worker applies for a job, furthermore it is changed related to the average productivity growth of the general skill group the household belongs to.

2. If a household is unemployed, then she receives an unemployment benefit from the government. This is a fixed percentage of households last earned wage.

3. Depending on the index share holdings, a household receives additionally to the labor income (respectively benefit payments) dividends, that are paid by firms to a clearing house. The clearing house distributes the total dividends to the holders of the index share.

4. The household receives interest payments from the bank on the deposits.

The total income is the sum of these different income types. At the end of the month a household has to pay income taxes on the total income.

3.6.2 Consumption and savings decision

At the beginning of period t, a consumer h decides on the budget $C^{exp}_{h,t}$ that she plans to spend. The consumer receives an income $I_{h,t}$, and has a total wealth $W_{h,t}$, consisting of money holdings and asset wealth in shares.

The consumer sets his consumption according to the following consumption rule

$$C^{exp}_{h,t} = I^{Mean}_{h,t} + \kappa \cdot (W_{h,t} - \Phi \cdot I^{Mean}_{h,t}), \tag{82}$$

where $I^{Mean}_{h,t}$ is the mean individual income of an agent over the last T periods and the parameter Φ is the target wealth/income ratio. This formulation is motivated by the "buffer stock" theory of consumption which is backed up by theoretical arguments and substantial empirical evidence (see Deaton (1991), Carroll and Summers (1991)). As argued in Carroll and Summers (1991) this theory, according to which liquidity constrained consumers facing risky income desire a buffer stock equal to a certain number of months income, has the potential to explain empirically observed correlation between income, savings and growth. The parameter Φ describes how large the targeted buffer is relative to income and κ indicates how sensitive the consumption reacts to deviations of the actual wealth/income ratio to the target level.

Each consumer shops once a week, but different households at different days. The monthly budget is equally split over the four weeks. Parts of the weekly budget that are not spent in a given week are rolled over to the consumption budget of the following week. This yields a weekly consumption budget $B^{cons}_{k,week_t}$ for each week in period t.

3.6.3 Financial asset allocation

Households decide their asset allocation after their consumption budget is determined. This may lead to a re-adjustment of their current asset portfolio.

Since we do not assume that capital markets are perfectly liquid, the agents are not ensured that they will be able to execute their transactions. There is the possibility of rationing, also on the financial markets.[4] Therefore we have to introduce a distinction between the anticipated and the realized values of variables such as the consumption budget and asset allocations.[5] The anticipated values are planned at the start of period t, while the realized values are known after the financial market is closed. The agents then know their asset transactions and the status of their money holdings after trading. Only then will the consumption budget become firm and

[4]The problem of market illiquidity can be softened somewhat by introducing a market maker into the model, but this does not solve the issue entirely. It could still occur that this market maker is not willing to take the counter-party risk is all market participants are trying to sell.

[5]The so called ex ante/ex post analysis of the Stockholm School.

they visit the market for consumption goods where they spent their actual consumption budget which may deviate from the *ex ante* planned budget.

Households first determine their anticipated consumption budget C^e_{ht} based on their current total wealth W_{ht} at the start of the period, consisting of liquid money holdings and the value of the asset portfolio. Subtracting this planned but not yet realized consumption budget from total wealth the remaining *'asset budget'* is allocated to assets.

In a second step, the households determine the asset allocation between the risky asset (the market index) and the risk-free asset (bank deposits). Given the planned asset budget X^e_{ht}, the total value allocated to the risky asset is $A^e_{ht} = \pi_{ht} X^e_{ht}$, where $0 \leq \pi_{ht} \leq 1$ is the proportion to invest, and $M^e_{ht} = (1 - \pi_{ht}) X^e_{ht}$ are anticipated money holdings.

Wealth-based asset allocation. To derive the household's orders for shares in the market index we consider the literature on artificial financial markets that use the mean-variance maximization framework of the capital asset pricing model (Chiarella and He (2000, 2001), Hommes (2006)).

A wealth-based version of the mean-variance maximization framework is provided in Chiarella and He (2000, 2001), who consider an investor who faces a given price p_t, has wealth W_{ht} and believes the asset return is conditionally normally distributed with mean $E_{ht}[\rho_{t+1}]$ and variance $V_{ht}[\rho_{t+1}]$. The investor chooses a proportion π_{ht} of her wealth to invest in the risky asset, so as to maximize the expected utility of the wealth at time $t + 1$:

$$\max_{\pi_{ht}} E_{ht}[U(W_{h,t+1})] \tag{83}$$

The optimal investment proportion in the risky asset is then given by:

$$\pi_{ht} = p_t \frac{E_{ht}[p_{t+1} + y_{t+1} - Rp_t]}{V_{ht}[p_{t+1} + y_{t+1} - Rp_t]} = p_t z_t. \tag{84}$$

If we assume identical beliefs about asset prices the agents would all have the same optimal proportion to invest π_t and this would lead, under identical current portfolios, to exactly the same investment decision.[6] This would result in all traders to be on the same side of the market, and they would all trade only with a market maker who takes the opposite side of the transactions. However, since we want households to mostly trade amongst each other we want to prevent such a situation.

In order to obtain a model in which households have heterogeneous beliefs about asset prices, at this point we need a model to determine the subjective beliefs about the values of E_{ht} and V_{ht}.

Simplifying assumption. Instead of assuming that the proportion to invest in the risky asset is related to the risk-weighted excess return, we assume that π_{ht} is a uniformly distributed random variable.

Such an assumption is sufficient to obtain different values for π_t, and so to ensure that households are on opposite sides of the market. The formulation captures in a reduced form the heterogeneity of traders beliefs leading to different allocations between the risky and the riskless asset.

[6] It is not uncommon that the initialization of asset portfolios is identical for all traders. This is due to the idea that we want to start with a uniform wealth distribution, and let the model generate a skewed wealth distribution instead of starting with a skewed distribution in the first place.

This results in the following sequence of equations:

$$X_{ht} = M_{ht-1} + A_{ht-1} - C_{ht-1} \tag{85}$$
$$W_{ht} = M_{ht} + A_{ht} \tag{86}$$
$$X^e_{ht} = W_{ht} - C^e_{ht} \tag{87}$$
$$A^e_{ht} = \pi_{ht} X^e_{ht} \tag{88}$$
$$M^e_{ht} = (1 - \pi_{ht}) X^e_{ht} \tag{89}$$

where:

X_{ht}: Actual asset allocation at end of period $t-1$, start of period t.

W_{ht}: Total wealth of agent h at the start of period t.

C^e_{ht}: Planned consumption budget.

X^e_{ht}: Planned asset budget to be allocated.

A^e_{ht}: Asset allocation to risky asset, planned at time t.

M^e_{ht}: Planned money holdings, planned at time t.

$\Delta X_{ht} = X^e_{ht} - X_{ht-1}$: Planned change in asset allocation.

$\Delta A_{ht} = A_{ht} - A_{ht-1}$: Actual change in risky assets.

$\Delta M_{ht} = M_{ht} - M_{ht-1}$: Actual change in money holdings.

Since the change in the total asset allocation can have either sign, we capture both investment and disinvestment. Overall we should distinguish several cases (see Table 6):

1. $\Delta X_{ht} > 0$: The asset budget is anticipated to increase with respect to the current state (note that $\Delta X_{ht} = \Delta A^e_{ht} + \Delta M^e_{ht}$). Subcases are:
   (a) $\Delta A_{ht} > 0$: Buy risky assets and withdraw money. This is a net investor who wants to invest in the risky asset. Some money from bank deposits are reallocated to the asset portfolio.
   (b) $\Delta A_{ht} \leq 0$, $\Delta M_{ht} > 0$: Sell risky assets and deposit money. This is a seller who wants to liquify risky assets to reallocate wealth from the portfolio to deposits, possibly without affecting the total allocation to assets.
   (c) $\Delta C_{ht} < 0$: Buy risky assets and deposit money.
2. $\Delta X_{ht} < 0$: The asset budget is anticipated to decrease with respect to the current state. This can be achieved either through a decrease in both terms or by an overall decrease and a reallocation between the risky and risk-free asset. Subcases are:
   (a) $\Delta A_{ht} < 0$ and $\Delta M_{ht} \leq 0$: a seller plans to withdraw money from his bank account and sell risky assets to consume more.
   (b) $\Delta A_{ht} < 0$ and $\Delta M_{ht} > 0$: a seller wants to increase money holdings and consumption by selling risky assets.

3. $C_{ht} \geq M_{ht}$: A special case occurs if the agent's consumption budget is larger than current money holdings and the household is rationed so it cannot liquify its assets. There are still risky assets in the portfolio to finance the difference $C_{ht} - M_{ht}$, but now it turns out that she has to downscale the consumption budget. We assume that the consumption budget is then set equal to the actual payment account, and the household completely depletes its current money holdings.

Table 6: Household asset allocation.

ΔC^e	ΔA^e	ΔM^e	Description
$+$	$+$	$-$	Consume more, buy assets, withdraw money (reallocate)
$+$	$-$	$+$	Consume more, sell assets, deposit money (reallocate)
$+$	$-$	$-$	Consume more, sell assets, withdraw money (reduce assets)
0	$+$	$-$	buy assets, withdraw money (reallocate)
0	$-$	$+$	sell assets, deposit money (reallocate)
$-$	$+$	$+$	Consume less, buy assets,, deposit money (increase assets)
$-$	$-$	$+$	Consume less, sell assets, deposit money (reallocate)
$-$	$+$	$-$	Consume less, buy assets, withdraw money (reallocate)

3.6.4 Consumption choice

The consumer collects information about the range of goods provided. He receives information about prices and inventories. In the Marketing literature it is standard to describe individual consumption decisions using multi-nomial logit models (see e.g. Malhotra ((1984)), and also substantial empricial evidence has been collected concerning the intensity of choice with repsect to different product features in different markets (see Krishnamruthi and Raj (1988)). We assume that a consumer's decision which good to buy is random, where purchasing probabilities are based on the values she attaches to the different choices she is aware of. Denote by $G_{h,week_t}$ the set of producers whose goods consumer h has sampled in week $week_t$ of period t and where a positive stock is available at the attended mall. As in our setup there are no quality differences between consumer goods and we also do not explicitly take account of horizontal product differentiation, choice probabilities depend solely on prices. The value of consumption good $i \in G_{h,week_t}$ is then simply given by

$$v_h(p_{i,t}) = -\ln(p_{i,t}). \tag{90}$$

The consumer selects one good $i \in G_{h,week_t}$, where the selection probability for i reads

$$Prob_{h,i,t} = \frac{\exp\left(-\gamma^C v_h(p_{i,t})\right)}{\sum_{i' \in G_{h,week_t}} \exp(-\gamma^C v_h\left(p_{i',t}\right))}. \tag{91}$$

Thus, consumers prefer cheaper products and the intensity of competition in the market is parameterized by γ^C. Once the consumer has selected a good she spends her entire budget $C^{exp}_{h,week_t}$ for that good if the stock at the mall is sufficiently large. In case the consumer cannot spend all her budget on the product selected first, she spends as much as possible, removes that product from the list $G_{h,week_t}$, updates the logit values and selects another product to spend the remaining consumption budget there. If she is rationed again, she spends as much as possible on the second selected product, rolls over the remaining budget to the following week and finishes the visit to the mall.

3.6.5 Specific Skill Adaptation

Each worker h has two dimensions of human capital endowments namely an exogenously given general skill level b_h^{gen} and an endogenously increasing specific skill level $b_{h,t}$. General skills can be interpreted as formal qualification or general embodied abilities while specific skills are experiences or abilities obtained on-the-job reflecting the productivity of each worker. There exist five general skill levels, described by different values of b^{gen}, i.e. $b^{gen} \in \{1, 2, 3, 4, 5\}$. $b^{gen} = 1$ is the lowest general skill level and $b^{gen} = 5$ the highest. Another distinction is that general skills are observable by firms in the hiring process while specific skills are not. They become observable during the production process. The acquisition of specific skills in the production is faster for higher general skill levels. Formally, the workers increase the specific skills over time during production by a learning process. The speed of learning depends on the general skill level b_h^{gen} of the worker h and the average quality of the technology $A_{i,t}$ used by employer i.

$$b_{h,t+1} = b_{h,t} + \chi(b_h^{gen}) \cdot \max[0, A_{i,t} - b_{h,t}] \tag{92}$$

Here $b_{h,t}$ are the specific skills of worker h in period t and $\chi(b_h^{gen})$ increases with general skills b_h^{gen} and $0 < \chi(b_h^{gen}) < 1$.

3.6.6 Labor supply

Each worker h offers one unit of labor per month (20 days). If a worker is employed he receives a wage $w_{h,t}$ and uses the unit of labor in one firm. He is not able to divide the unit of labor and to distribute it to multiple firms. If the worker is unemployed he receives unemployment benefits from the government and is actively searching for a new job on a given day with probability p^{search}. This probability is determined by

$$p^{search} = \frac{\eta^{month}}{\eta^{day} \cdot 20} \tag{93}$$

where η^{month} is the number of applications a household is allowed to send per month and ηday per day. These parameters can be used to steer the intensity of search. An unemployed worker takes the wage offers posted by searching firms into consideration and compares them with his reservation wage $w_{h,t}^R$. An unemployed worker will only apply at a firm that makes a wage offer which is higher than his reservation wage

$$w_{i,t,g}^O > w_{h,t}^R. \tag{94}$$

When a worker applies he sends information about his general skill level to the firm. In case the firm is in a foreign region the unemployed worker also takes fixed commuting costs $comm$ into consideration

$$w_{i,t,g}^O > w_{h,t}^R - comm. \tag{95}$$

The level of the reservation wage is determined by the current wage $w_{h,t}^R = w_{h,t}$ if the worker is employed, and in case of an unemployed worker by his adjusted past wage. That is an unemployed worker will reduce his reservation wage with the duration of unemployment. If an unemployed worker did not find a job although he was actively searching he reduces his reservation wage by a fraction ψ, that is

$$w_{h,t+1}^R = (1 - \psi) w_{h,t}^R. \tag{96}$$

There exists a lower bound to the reservation wage w^R_{min} which is the level of unemployment benefits. If the unemployed worker receives one or more job offers he accepts the job offer with the highest wage offer. In case he does not receive any job offers he remains unemployed.

3.7 Government

The government mainly has redistributive functions. It levies an income tax on households and a corporate tax on firms. Furthermore it collects social security contributions. Outlays of the government include unemployment benefit payments and various transfers and subsidies to firms and households which may be switched on or off depending on the kind of policy experiment one wishes to run. For example, the model allows for subidies to be paid for training which raises the general skills of workers. These subsidies may be paid to firms residing in a specific region or to all firms populating the model.

The government calculates its budget on the basis of naive expectations at the start of every year. It projects the revenues and expenditures by multiplying last year's income and expenditure with an anticipated growth rate, respectively. Policies are adjusted at the end of every year such that the projected budget is covered. Policies are not altered for the entire upcoming year.

The Government computes its budget once per month. Should the government run a budget surplus it is deposited at the central bank. For simplicity we assume that a deficit is financed through monetization, i.e. the Central Bank creates fiat money.

3.8 Eurostat

The Eurostat agent collects and aggregates microvariables to construct macrovariables. These variables are computed online whenever there is a need for any of the agents to know about these macrovariables.

4 Market mechanisms

4.1 Credit market interaction

Banks accept credit requests by taking into account individual risks of the applicant and risks of their loan portfolio according to stylized Basel II standards. If the credit request is refused, or not fully accepted, the firm has to reduce its planned production quantity up to the point where planned expenditures can be financed.

If the firm is credit constrained to such extent that it is not able to pay the financial commitments it goes out of business. It goes bankrupt if the firm's equity becomes negative. In case of bankruptcy employees loose their jobs, it writes off a fraction of its debt and stays idle for a certain period before it becomes active again.

4.2 Labour market interaction

According to the procedures described in the previous sections consumption goods producers review once a month whether to post vacancies for production workers. Job seekers check for vacancies. The matching between vacancies and job seekers works in the following way:

Step 1: The firms post vacancies including wage offers for each general skill group.

Step 2: Every job seeker extracts from the list of vacancies those postings to which he fits in terms of his reservation wage $w^R_{h,t}$ net of commuting costs that may arise if he applies for a job in a region where he does not live. He sends an exogenous determined number of applications to randomly chosen firms.

Step 3: If the number of applicants is smaller or equal to the number of vacancies the firms send job offers to every applicant. If the number of applicants is higher than the number of vacancies firms send job offers to as many applicants as they have vacancies to fill. Applicants with higher general skill levels b^{gen}_h are more likely to receive a job offer.

Step 4: Each worker ranks the incoming job offers according to the wages net of commuting costs that may arise if he was to accept a job in the region where he does not live. Each worker accepts the highest ranked job offer at the advertised wage rate. After acceptance a worker ignores all other job offers and outstanding applications.

Step 5: Vacancy lists are adjusted for filled jobs and the labor force is adjusted for new employees.

Step 6: If the number of unfilled vacancies exceeds some threshold the firm raises the base wage offer. If an unemployed job seeker did not find a job he reduces his reservation wage. Go to step 1.

This cycle is aborted after two iterations even if not all firms may have satisfied their demand for labor. This might lead to rationing of firms on the labor market and therefore to deviations of actual output quantities from the planned quantities.

4.3 Consumption market interaction

The consumption goods market is modeled as a decentralized goods market. Each local market is represented by an outlet mall at which the consumption goods producers can offer and sell their products to their customers. While firms are free to serve all malls regardless their spatial proximity, households are limited in choosing their market places by allowing them only to visit that mall which is located in the direct neighborhood.

Households go shopping once a week and then they try to spend their entire weekly consumption budget for one good. At the beginning of their shopping procedure they get information about the prices of all available goods at the mall, but they get no information about the available quantities. The decision on which good to buy is described in Section 3.6.4.

Households have asynchronized shopping days and thus on each day of the months there may be shopping activities in the mall. The consumption requests for the different goods are collected by the mall and, if the total demand for one good exceeds its mall inventory level then the mall has to ration the demand. In this case the mall sets a rationing quota corresponding to the percentage of the total demand that can be satisfied with the available goods. Each households receives then the indicated percentage of her requested consumption.

After the shopping activity rationed households have still budget at hand. Those households have the opportunity to spend the remaining budget for another good in a second shopping loop. In this case the shopping process is repeated as described above.

The production of the consumption goods firm follows a fixed time schedule with fix production and delivery dates. Even if the mall stock is completely sold out it can only be refilled at the fixed delivery date. Consequently, all the demand that exceeds the expected value of the monthly sales and the additional buffer can not be satisfied.

4.4 Financial market interaction

The main purpose of the financial market model is to close the macroeconomic simulation model and provide the linkage between the real and financial sector. Center to the financial market modeling is a clearinghouse which organizes the trade of a market index share. The aim is therefore not to model a real stock market exchange in full detail, but to approximate it in order to have firms with publicly traded shares.

There are two assets: risk-free deposits at a bank, and a risky asset which is a stock market index, composed of the equity shares of the individual firms.

Firms have a fixed number of outstanding equity shares, but do not issue new shares. Firms pay out a dividend on their shares that depends on their profits. Banks also have a fixed number of outstanding equity shares that can be traded and pay a dividend. Households can trade shares of the market index but not the shares of the individual firms (see Section 3.6.3 for households asset allocation). The stock market index pays a dividend, such that firm profits enter back into the economy.

The clearinghouse aggregates the total dividends during the calendar month and computes the dividend that is paid on the market index shares, based on the information it received from the firms and banks on the total dividend payments.

4.4.1 Asset pricing mechanism for the market index

We do not assume that the price of the market index at a given point in time leads to market clearing. Furthermore, there is no market maker, and no limit-order book. This is surely a simplifying assumption, and it is made for the reason that we do not need a fully fledged financial market model in order to study the macroeconomic dynamics of the real side of the economy. All that is needed is a mechanism to close the model, to redistribute the profits of the firms back into the economy through dividends, and to allow households to trade the ownership rights of the capital stock embedded in the firms.

The price of a share in the market index depends on the asset trading of households and is computed by the clearinghouse. No equilibrium pricing is used, but a cautious price adjustment process with ceilings and floors on the price growth rate (this reflects so called circuit breakers):

$$\Pi = \left(\frac{D_t}{S_t}\right)^{\lambda}, \tag{97}$$

$$p_{t+1} = \begin{cases} rp_t, & \Pi \leq r, \\ \Pi p_t, & r \leq \Pi \leq R, \\ Rp_t, & R \leq \Pi. \end{cases} \tag{98}$$

where $\lambda > 0$ is a price adjustment speed, and r and R are lower and upper bounds on the price growth rate, respectively.

4.4.2 Asset allocation mechanism for the market index

The clearinghouse determines transactions in the market index shares using a simple proportional rationing mechanism that is based on the ratio between demand and supply. Given a monetary value z_h that an investor wants to invest, $\lfloor z_h/p_t \rfloor$ are the corresponding units of shares to trade, where $\lfloor x \rfloor$ is the floor operator defined by the largest integer not greater than x.

Let s_{ht} represent the individual selling orders (in negative units), d_{ht} are the individual buying orders (in positive units), and let D_t and S_t be the aggregate demand and supply (in positive units), resp.

$$d_{ht} = \begin{cases} \lfloor z_h/p_t \rfloor, & z_h > 0 \\ 0, & z_h \leq 0. \end{cases} \tag{99}$$

$$s_{ht} = \begin{cases} \lfloor z_h/p_t \rfloor, & z_h < 0 \\ 0, & z_h \geq 0. \end{cases} \tag{100}$$

$$D_t = \sum_h d_{ht}, \quad S_t = -\sum_h s_{ht}. \tag{101}$$

The rationed demand/buy orders B_{ht} and the rationed supply/sell orders S_{ht} are given by

$$B_{ht} = \begin{cases} d_{ht}, & \left(\frac{D_t}{S_t}\right) \leq 1, \\ \left\lfloor \left(\frac{D_t}{S_t}\right)^{-1} d_{ht} \right\rfloor, & \left(\frac{D_t}{S_t}\right) > 1. \end{cases} \tag{102}$$

$$S_{ht} = \begin{cases} s_{ht}, & \left(\frac{D_t}{S_t}\right) \geq 1, \\ \left\lfloor \left(\frac{D_t}{S_t}\right) s_{ht} \right\rfloor, & \left(\frac{D_t}{S_t}\right) < 1. \end{cases} \tag{103}$$

We assume there is a zero supply of outside shares, so all buy and sell transactions respect the total units available. If some trader wants to invest a lot of money and buy many units at the current price, these units simultaneously have to be offered by some sellers. As a conservation rule, we can check that the total units of shares in the index are conserved:

$$\sum B_{ht} + \sum S_{ht} = 0. \tag{104}$$

5 Verification

An important part of the testing and verification process of the Eurace@Unibi Model involves the internal consistency of model by looking into the stocks and flows. Stock-flow (SFC) consistent models are defined as:

> "[...] models that identify economic agents with the main social categories/institutional sectors of actual capitalist economies – thoroughly describe these agents' short-period behaviors and consistently model the 'period by period' balance sheet dynamics implied by the latter." (Macedo e Silva and Dos Santos (2008, p. 2))

Accordingly it is checked that all monetary flows are accounted for, and that all changes to stock variables are consistent with the flows. This is accomplished by tracking the time evolution of the balance sheets across the different sectors of the economy. A Social Accounting Matrix (SAM) is constructed that contains all the monetary flows and changes to the balance sheet between the beginning and end of an accounting period. The SAM consists of a double-entry accounting system in which each flow comes from somewhere and goes to somewhere. It shows how the balance sheets of the different economic sectors (agents) are interlinked, and it also shows how the period-by-period balance sheets change over time. Such an accounting system

at the macro level provides the user with accounting identities that should always hold. That these identities hold can be tested by an external invariant detector such as Daikon.

SFC provides a solid and economically well-founded methodology to test the consistency of the model and it increases the credibility that can be attached to the model's results. Thereby it is not only part of the testing and verification procedure, but is also part of the accreditation process. It will helps to raise the acceptability and trust in the model.

6 Simulation

6.1 Parametrization

The model hosts a considerable number of parameters. The most important ones are summarized in table 7. Values of all parameters are chosen to reflect empirical evidence whenever possible. The ratio of the number of households (workers) and firms matches mean firm sizes to be observed in Europe. The innovation probability is chosen to reflect estimates approximating shifts of the technological frontier. Comparable to data reported in Vandenbussche et al. (2006) our calibration yields a growth rate of the technological frontier of around 6% per year if skills were sufficient to fully exploit technological innovations. Wage updates are calibrated to match wage growth in Germany during the decade of full employment in the sixties. The parameter value for the adjustment of the reservation wage is based on reported wage losses of approximately 17% after spells of unemployment in Germany (see Burda and Mertens, 2001), and an average duration of unemployment of 30 weeks. As a proxy for the reservation wage we make use of the net replacement rates of unemployment benefit schemes in OECD countries (see OECD, 2004). For the marginal propensity to save we chose 0.1 which is close to the savings rate in Germany in previous years. The intensity of the consumer choice stems from estimated multi-nominal logit models of brand selection. Estimates based on market data (Krishnamruthi and Raj, 1988) provide a lower bound of 6.

Table 7: List of parameters.

Symbol	**Name**	**Default**	**Range**
Agents			
	Households	1600	
	Firms	80	
	Investment good producers	1	
	Banks	2	
	Central Bank	1	
	Government	1	
Consumption goods			
tax	Income tax rate	0.05	$0.02 - 0.12$ (0.02)
u	Unemployment benefit pct	0.70	$0.60 - 1.00$ (0.10)
κ	Marginal propensity to save	0.1	
Φ	Target wealth/income ratio	16.67	
δ	Capital depreciation rate	0.01	
χ	Service level for the expected demand	0.8	

Continued on next page

Table 7 – continued from previous page

Symbol	Name	Default	Range
γ^C	Intensity of consumer choice	12	6 – 18
ρ	Discount rate	0.02	
Investment goods			
	Initial capital price	20	
Δq^{inv}	Technological progress	0.025	0.00 – 0.05 (0.0125)
λ	Bargaining power of the capital goods producer	0.5	0.0 – 1.0
γ^v	Logit parameter for vintage choice	30.0	
Credit market			
T^{Loan}	Debt repayment period	18	6 – 24 (6)
ω	Debt rescaling factor	0.30	0.0 – 0.8 (0.2)
r^c	ECB base rate	0.05	0.01 – 0.05 (0.01)
r^d	Deposit rate	0.045	0.01 – 0.05 (0.01)
Financial market			
d^f	Dividend payout ratio	0.70	0.1 – 0.7 (0.1)
$\bar{d}$	Threshold full dividends (firms)	0.5	
α^b	Basel capital requirement	10	4 – 10 (2)
β^b	Min. cash reserve ratio	0.10	
λ^{fin}	Parameter price adjustment rule	1.0	
π^{min}	Ceiling on price growth rate	1.10	
π^{max}	Floor on price decrease rate	0.90	
Labour market			
φ	Wage update	0.01	
ψ	Wage reservation update	0.01	
η^{month}	Number applications	5	
η^{day}	Applications per day	3	
ϱ^{down}	Lower bound firing	0	
ϱ^{up}	Upper bound firing	0.1	
γ^{gen}	Logit parameter general skills	0.5	

6.2 Initialization

In general there are several considerations that constrain the initialization of the agent's state variables. First and foremost, we cannot initialize the variables completely at random. This would violate the internal logic of the model, since in order to obtain a working simulation we have to initialize the agent's balance sheets according to the criterion of stock-flow consistency. This means that we are constrained to set the initial values such that the balance sheet relationships between agents hold. If the balance sheets would be inconsistent from the start they would remain so throughout the entire simulation.

The second consideration is that we start with reasonable or plausible values. This is in order to alleviate the initial transient effects that any initialization invariably has. In our experience large path dependencies can be generated by such initial transients, so it is important to carefully consider the interdependencies between the initial values.

- *capital price*: The initial capital price is assumed to be in a fixed relation to the initial

wage that is on average paid in the economy. Even if the IG firm does not employ workers it has a memory variable *wage offer* that is used only for the initialization.

- *unit costs*: The price setting of the IG firm is a combination of value and cost based pricing. For the cost based price component the IG firm takes virtual unit costs into account, i.e. a variable called *unit costs* that is a proxy for the costs which would arise in the production process. Since the costs usually change over time (mainly due to increasing labor costs) this change has to be incorporated in the evolution of the unit costs. In order to have a stable capital goods price in the first months, the unit costs have to be initialized at the same level as the initial capital goods price.

- *output*: We set the output of a firm at a level such that the total labor demand that is needed for producing the cumulated output would correspond to full employment.

- *total units capital stock*: The capital stock is set to have a sufficient capital stock in order that the initial production quantity can be produced without additional capital investments.

- *total value capital stock*: This is an asset on the balance sheet of the firm.

- *payment account*: The value of the firm's payment account is set to equal the value of its capital stock, such that the firm has sufficient liquidity in the first month to start repaying the initial loan that was inherited from historical investments.

- *total value local inventory*: The firm has no initial inventory stock.

- *initial loan*: We start the firms with an initial loan in order to approximate plausible leverage ratios. This will alleviate initial transient effects. The initial loan is set according to a constant leverage ratio of 2.0. This implies that the initial loan is (2/3) of total assets and equity is (1/3) of total assets.

- *capital financing per month*: We assume that the firm has invested in capital during its history before day 0 (the start of the simulation). The investments are exactly that amount which is necessary to compensate for the monthly depreciation of capital such that the capital stock remains constant. In order to stabilize the simulation with regard to bankruptcies at the beginning we deviate from the usual assumption that the loan obtained for the investments has to be repaid in the standard repayment period of a loan. Instead we allow the initial loan to be repaid in twice the length of time (24 months).

- *employee firm id*: We assume all households are initially unemployed.

- *wage reservation*: The reservation wage is set equal to the firms wage offer, such that households accept job offers in the first month.

- *mean net income*: Mean net income is set equal to the reservation wage.

- *payment account*: Households have an initial payment account equal to 15 monthly wages, to represent a plausible savings buffer.

- *assets owned*: The households asset portfolio is scaled to yield a wealth level that is reasonable. Each household is endowed with an equal number of index shares, with a price such that the total value of the initial portfolio is 10 (each household has risky-asset wealth equal to 10 monthly mean wages).

- *wealth*: Households' initial total wealth consists of liquid money holdings in the payment account and illiquid asset holdings. Together with the payment account of 15 monthly wages the initial total wealth of each household is 25.
- *number of shares*: The number of index shares is scaled to the total number of firms and households.
- *weight*: The weight of the firms in the index share is uniform. This is needed to compute the dividend per index share from the total dividend payment of the firms.

6.3 Simulation output

In order to illustrate the simulation output that the model is able to generate, some plots generated in standard parameter scenarios are given, and are briefly discussed below. Due to the bottom-up approach of agent-based models, also time series at more disaggregated levels can be investigated. In principle, it is possible to screen the behavior or a single agent over time, i.e. a firm or a household if necessary for the analysis of the topic being investigated. We first briefly discuss the dynamics of the main economic variables on the macro level and point out some business cycle properties. Corresponding dynamics on the different types of markets are also illustrated. Furthermore, we demonstrate that changes in the parametrization of the model, which have clear economic interpretation, generate scenarios where substantial crashes of the economy followed by recoveries occur. Finally, distributional properties of the simulation results are shown from a static and a dynamic perspective. As will become clear many of the qualitative properties of the generated simulation output match documented empirical stylized facts.

6.3.1 Macro variables and business cycle properties

Figure 1 shows the main macro variables. The figure is based on a representative single simulation run rather than the mean or median of a batch of simulations, since we do not want to average out the business cycle properties of the generated time series. Simulations have been run for 500 months, but in most plots we omit the first 100 months since the initial transition period is strongly influenced by the initialization and not representative of the medium and long run features of the model.

GDP dynamics is characterized by persistent growth with a business cycle component around the trend. The growth rate fluctuates in a range between -2 and +3 percent and shows a certain degree of persistence. There is persistent inflation in the economy, where the inflation rate fluctuates in a corridor between 0-2% with a mean of 1.5%. Also unemployment fluctuates, where its range is between 10-20%. As pointed out above, it is not the aim of this excercise to quantitatively match the time series of actual economies. Rather, we want to demonstrate that the model generates outcomes that are qualitatively consistent with the typical properties of macroeconomic time series. In order to do so, we present in figure 2 the bandpass filtered time series of output and several other variables, where the parameters of the bandpass filter are set to the standard values used in the analysis of empirical time series (see e.g. Stock and Watson (1999)). The business cycles can be clearly seen in the filtered time series, where it has to be recognized that the cycles seem to be rather short compared to most of the empirical data. Considering the co-movement between output and different key variables along the business cycle, we observe that consumption, investment and employment is strongly pro-cyclical with

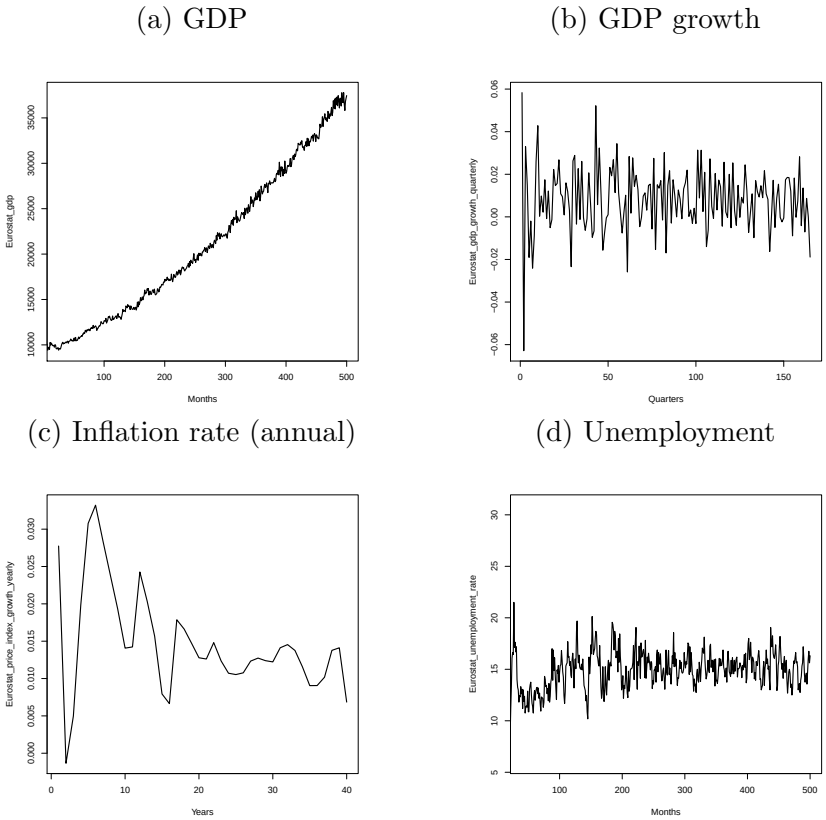


Figure 1: Macro variables in the baseline scenario. $\gamma^C = 12$.

almost no lag relative to output fluctuations. The amplitude of fluctuations of employment closely matches that of output (Figure 2 (c)), whereas the amplitude of consumption is smaller (Figure 2 (a)) and that of investment is larger (Figure 2 (b)) compared to output. All these observations match actual macroeconomic data, see e.g. figures 3.10, 3.15 and 3.3 in Stock and Watson (1999). Furthermore, total credit moves pro-cyclically (see figure 2d), which shows that private debt expands during booms and contracts during a decline. This indicates that in our stable benchmark scenario the primary reason for firms to increase debt is to finance expansion of production rather than to re-finance already existing debt.

Wages move counter-cyclically but with a much smaller amplitude than output (again, a similar observation can be found in Stock and Watson (1999), figure 3.43). Contrary to most standard macroeconomic models, the Eurace@Unibi model generates endogenously determined mark-ups which change over time. As can be seen in Figure 2 (f) mark-ups tend to move counter-cyclical. It has been argued in Blanchard (2008) that there is indeed empirical indication of counter-cyclical markups (see Bils and Chang (2000)) and that it is a wide open macroeconomic research question to provide sound theoretical explanations for the movement of mark-ups along the business cycle.

It should be pointed out that, although due to the stochastic nature of the innovation process the movement of the technological frontier is subject to stochastic influences, stochastic productivity shocks are not the driving force of the business cycles generated by the model. Detailed analysis of the dynamics of micro-level variables like demand expectations, investments, costs, prices, household income and consumption budget, show that the cycles are generated by the interplay of optimistic demand estimations of firms in the upswing, the resulting investment

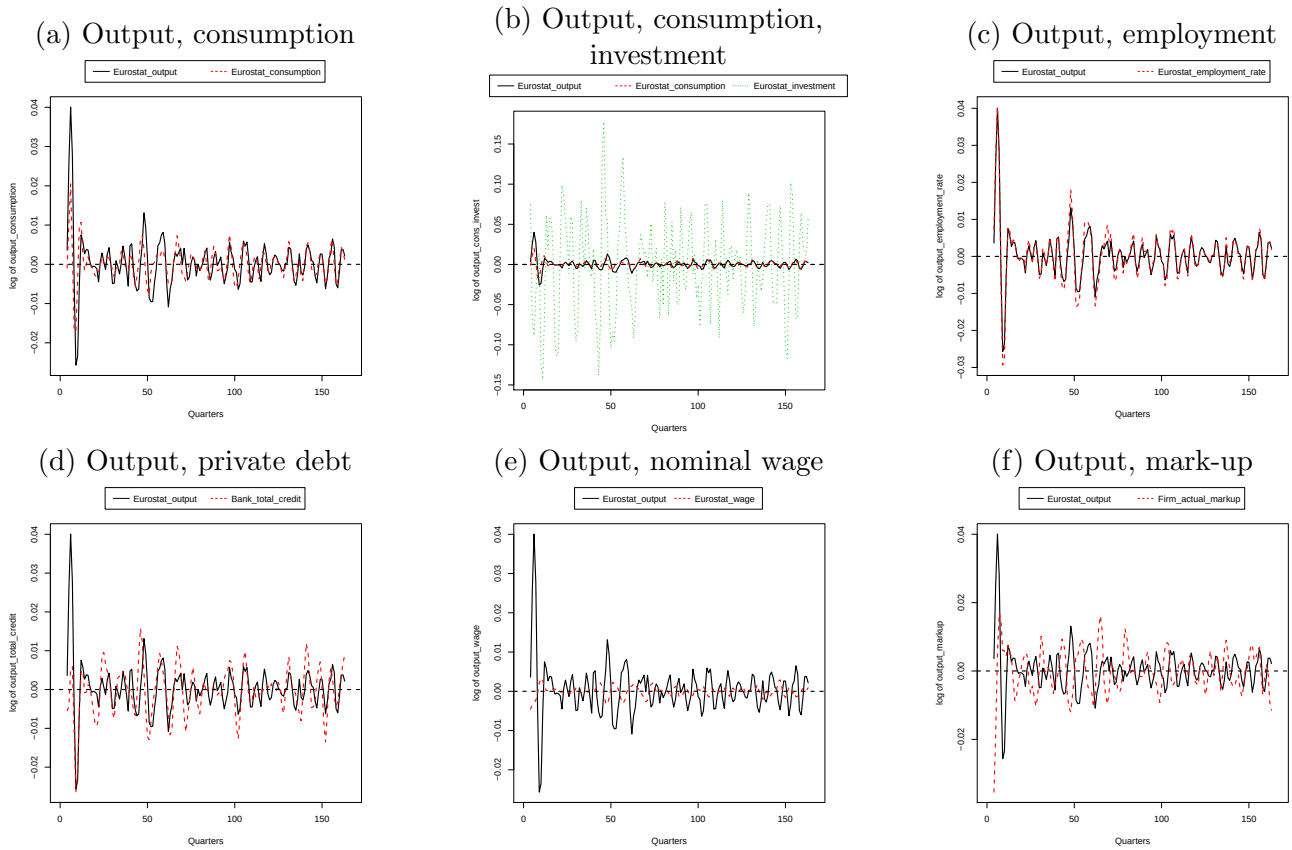


Figure 2: Bandpassed filtered timeseries.

patterns, the induced cost and price effects, the implications of these effects on sales, mark-ups and firm earnings and the reinforcement of such implications through the demand channel due to changes in the real consumption budget of households. A detailed analysis of these chains of effects is beyond the scope of this paper, but it should be noted that the heterogeneity of firms is an important factor in the determination of the cycles.

6.3.2 Dynamics of the different markets

Consumption goods market

Two important variables at the consumption goods market are the number of produced and the number of sold units of the consumption good. As seen in Figure 2 (a) consumption growth tends to evolve pro-cyclical relative to the output growth. Consumption and production are, however, not perfectly matched.

The actual consumption (or total sales of the consumption goods firms, respectively) is mainly driven by the consumption budget of households. Figure 3 (a) shows the time series of the real consumption. We can observe that consumption follows a clear growth trend but thereby it is highly volatile.

As described in Section 3.2.1, firms hold inventories at the local markets which are serviced in order to cover the upcoming local demand at an exogenously given service level. The actual production quantity of a firm is then determined by an interaction of the inventory stocks before delivery, the demand estimation, and the service level. A crucial role for the demand estimation plays thereby the price at which a good is offered as it is one of the main drivers of households'

purchasing decision.

The output of a firm can be defined as the inflow into the inventory stock whereas the sales/ real consumption can be seen as the outflow. It should be clear that, if the inflows and outflows are not balanced, the inventory level as the stock variable is not stationary. The change of the stock can be seen as an indicator whether or not the demand estimation matches the actual demand. Panel (b) shows the summed inventory stocks of all firms at the days before delivery. It illustrates two facts: first, in this simulation the total stock is never depleted completely. That means the sum of expected demands plus the safety buffers is sufficient to satisfy the total market demand. Secondly, the inventory stock seems to switch between levels either above or under the trend. This observation can be explained by an inaccurate demand estimation by the firms that occurs during an economic turnaround. The break of the economic trend leads thereby through the channel of backward looking expectations to a temporary forecast error regarding the short term economic development.

Panel (c) displays the capacity utilization of the total capital stock. Generally speaking, the capacity utilization is an important indicator for the business cycle. It has a relatively low level during recessions when the output is low and thus the firms underutilize their capital stock, whereas the level is closer to the maximum capacity during boom phases. In Panel (c) we observe a mean capacity utilization of 82%, with local tops and bottoms reflecting the booms and busts. Worth noting is that the capacity utilization seems to evolve almost completely inversely compared to the total stock in Panel (b) indicating the link between the stock holding and the business cycle.

(a) Sales (b) Inventory stocks (c) Capacity utilization

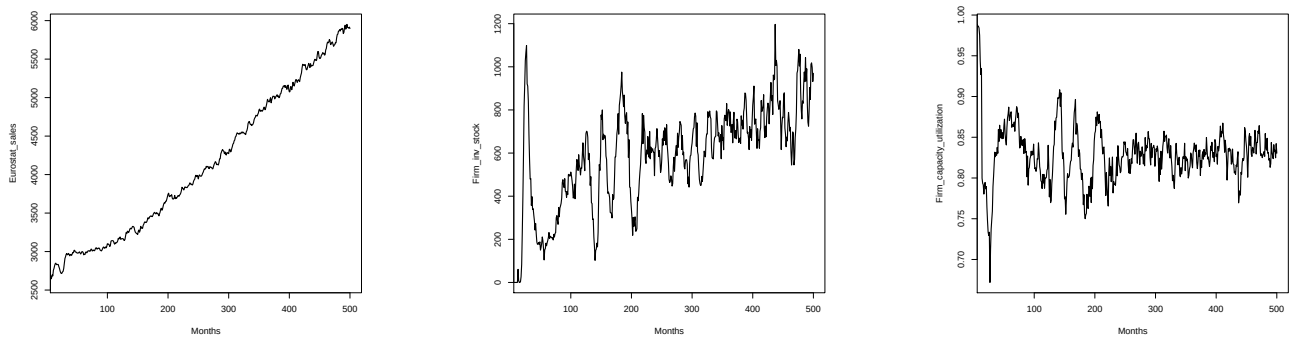


Figure 3: Consumption goods market dynamics. $\gamma^C = 12$.

Capital goods market

In Figure 4 we show several time series of key variables regarding the capital goods market. Panel (a) shows the development of the technological frontier defined as the productivity of the most recent vintage offered by the investment goods firm. This time series indicates that the frontier productivity is increasing over time, where the innovation intensity shows a varyingly strong concentration in different phases. This observation is a direct consequence of the endogenous innovation process as described in Section 3.5.1, where we assume that the probability of a successful innovation depends on the revenues of the investment goods firm. This link can also be seen when comparing the productivity growth with the nominal investments (Panel c). Here we observe a correlation between the nominal investments (i.e. the revenues of the investment goods firm) and the innovation rate as the endogenous innovation process suggests.

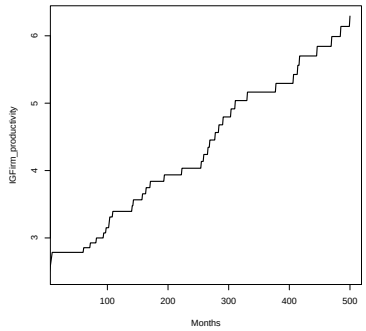
Panel (b) displays the price of the best practice technology. A general observation is that

the price of the most recent technology is also increasing over time. But by comparing the time series of the frontier technology (a) with the time series of the corresponding prices, we can observe that the capital goods price is not directly linked to the development of the productivity. Instead, the productivity jump induced by the introduction of a new vintage does not lead to a price jump at the same rate. This is because the investment goods producer anticipates that the buyers do not value the vintage at its technical productivity, since they cannot use the total productivity of a new vintage immediately due to the inertia of the specific skill adaptation (compare Section 3.5.2). Panel (b) indicates furthermore, that inbetween these innovation jumps there are also intermediate increases of the frontier price. This is due to the assumption that the (notional) unit costs of the investment goods producer increase, keeping pace with the economy-wide increase in labour wages.

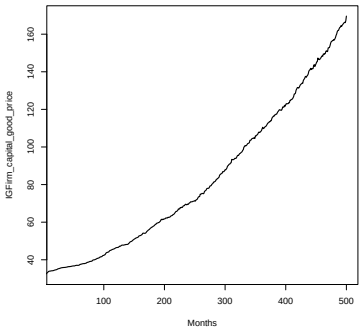
Panel (d) illustrates how the new technologies diffuse across the firms by showing the average productivity of the firms' capital stocks. Due to the vintage structure of the capital good, the capital stocks are composed of several vintages with different productivity levels, where the current composition depends on the history of previous investments. It should be clear that the average productivity of the capital stock of a single firm can only increase if the firm actually invests in a vintage whose productivity is above the average. Furthermore, it will never match the level of the best practice technology as long as there is older capital in operation. As a consequence there is a persistent gap between the actual productivity and the frontier meaning that the average technological productivity lags behind the technological frontier. Highly related with the capital productivity is the mean specific skill level of the workers who are employed by the firms (Panel e) since it is updated according to the growth in the firm specific technological productivity.

Panel (d) and (e) demonstrate that the productivity of the capital stock and the productivity of the workforce are both increasing over time; this implies that a firm could increase its output over time while keeping the inputs constant. This consequence of the technological progress can be observed in Panel (f) showing the total capital stock in the economy. Since the capital goods become more productive over time, the firms do not have to expand their capital stocks to produce more output. Hence total capital stock fluctuates around a constant mean.

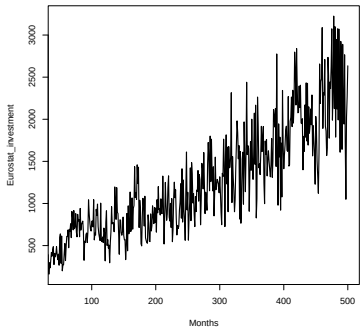
(a) Productivity of new technology



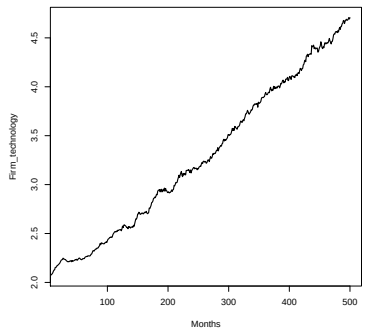
(b) Capital good price



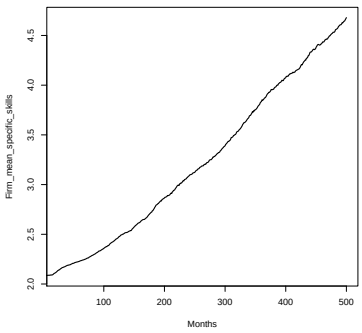
(c) Monthly investments



(d) Firm technology



(e) Firm mean specific skills



(f) Total capital stock

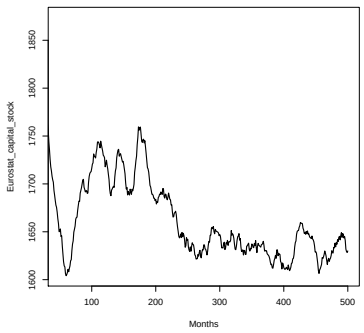


Figure 4: Capital goods market dynamics. Endogenous innovation process. $\gamma^C = 12$.

Labour market

Figure 5 (a-b) present the average number of employees per firm and the overall number of vacancies in the economy. If the firm has open vacancies after the labour market routine, which is the case in Panel (b) the firm increases its wage offer in order to attract more applicants. Figure 5 (c) illustrates the average wage offer in the economy. Since the increase of the wage offer does not depend on the number of unfilled vacancies but only on exceeding the threshold the wage offer increases monotonically.

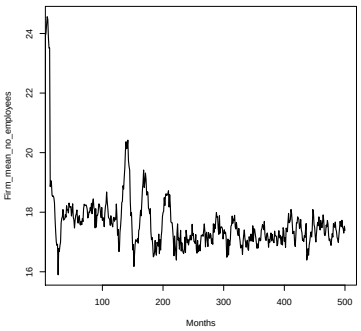
The firm's unit costs consist of several components, namely the wage bill, the investment costs in new capital, and the costs of previous financial commitments. Figure 5 (d-f) show the aggregated overall unit costs and two constituent parts; the labor costs per unit and the capital costs per unit. The plots indicate that the contribution of labor costs to the overall unit costs is an order of magnitude larger than the capital costs.

Total labour costs are increasing due to two reasons: the first is that the wage of incumbent workers is increased according to the average productivity growth in the economy. But the impact of the productivity driven wage update on the unit labour costs is negligible since the rise in wages is associated with a higher productivity and hence with the same but now more expensive labor force the firm can produce more output such that the overall effect on the unit costs might be neutral. The second driver of total labour costs is the wage the firms have to pay to newly hired workers. Those are paid the current wage offer, but since the wage offer is above the average effective wage paid in the firm, the mean wage grows stronger than the productivity when hiring new workers. Thus an increasing wage offer leads to higher unit labour costs.

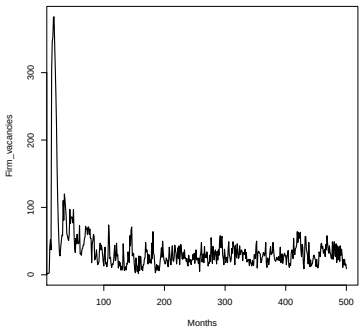
Panel (f) indicates that the capital costs are much more cyclical, which is due to the fact that capital investments, once they are spent, cannot be recovered. Thus, they become fix costs which, if treated as unit costs, vary with a changing production volume. Clearly the contribution of the unit capital costs to the overall unit costs as compared to the contribution of the labour costs is relatively small; consequently, the volatile capital costs cause only a slight volatility of the total unit costs.

The dynamics on the labor market illustrated above gives rise to qualitative features of the labor market data that correspond to well-known stylized facts. For example, as can be seen in Figure 6, a clearly downward sloping beveridge curve can be observed indicating a negative relationship between the unemployment rate and the number of vacancies.

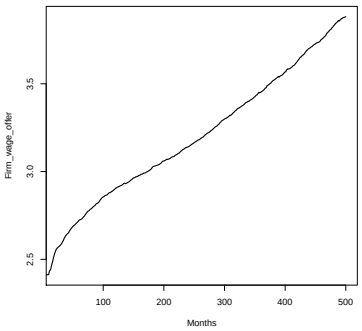
(a) Firm no. employees



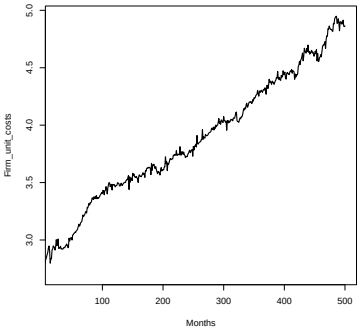
(b) Firm total vacancies



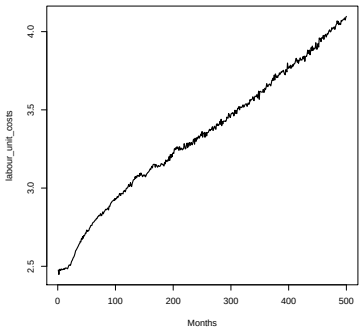
(c) Firm wage offer



(d) Firm unit costs



(e) Labour costs per unit (in aggregate)



(f) Calc capital costs per unit (in aggregate)

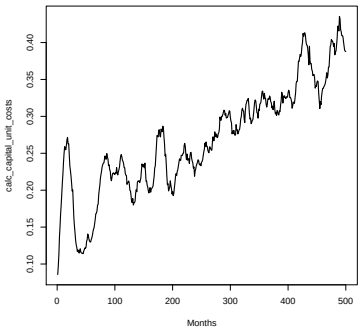


Figure 5: Labour market dynamics

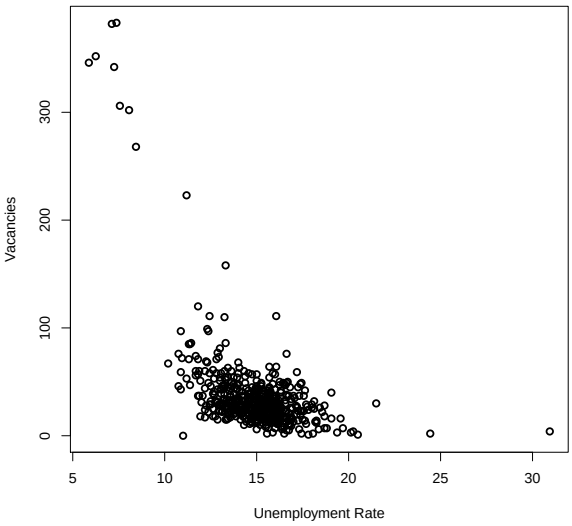


Figure 6: Beveridge curve

Credit market

In the baseline case the credit market dynamics is very simple due to the low number of firm bankruptcies. Figure 7 illustrates several credit market indicators. Panel (a) shows that profits are positive and growing, such that firms require external finance only to finance investments (hedge finance). They do not need new debt to pay for old debt (speculative or Ponzi finance). Panel (c) displays the average equity/asset ratio per firm as an indicator for the financial stability of the economy. In the baseline scenario the financial system is stable and the equity/asset ratio is slowly converging towards 0.5, which would imply firm debt is only halve of the asset value.

From the banks' point of view, the number of new loans provided each month is close to one per active firm (the number of active firms remains above 75 out of 80). The total credit provided is close to 3 times the GDP generated (Panel b). Panel (f) indicates that the banks' exposure to default risk is very low – just 20% of the total outstanding credit, and there are only few bankruptcies in the baseline case. These are visible as sharp downward drops in the bank exposure.

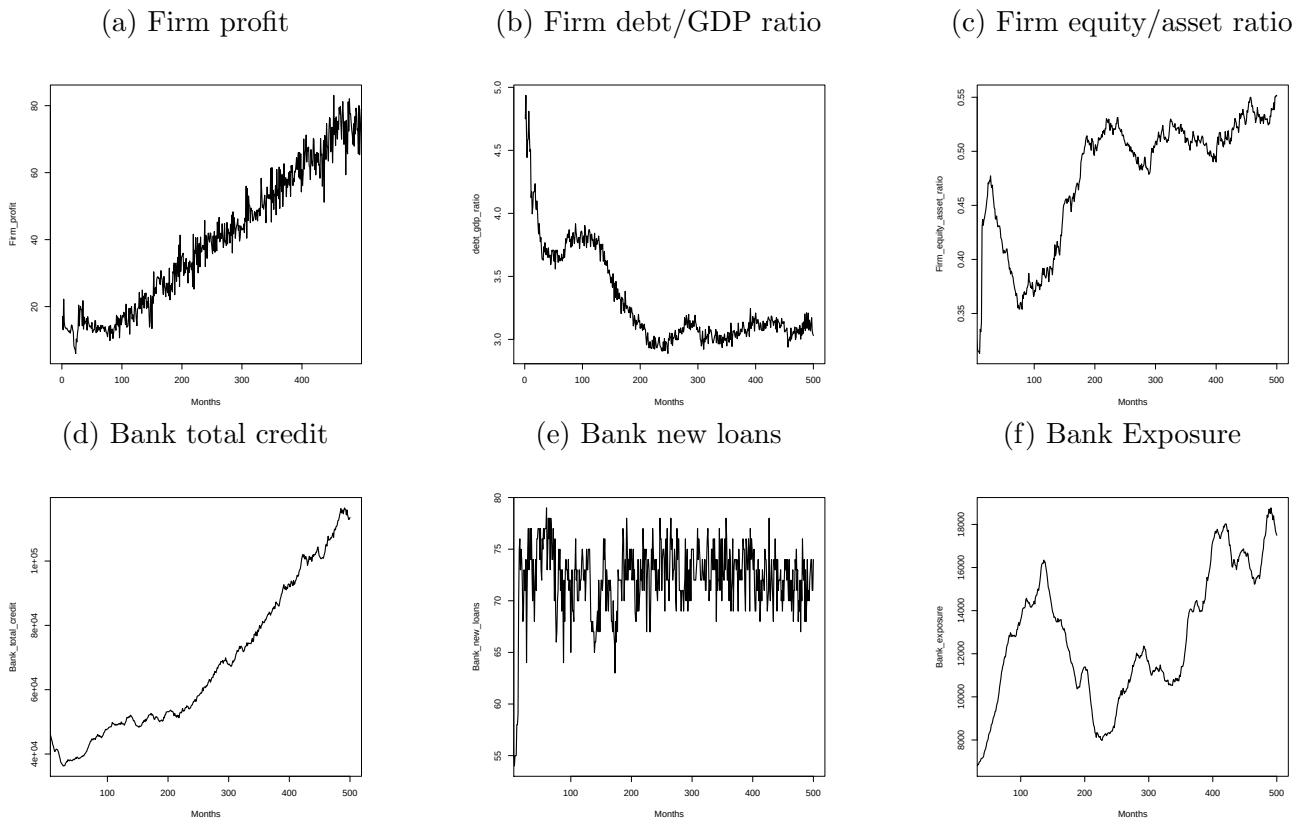


Figure 7: Credit market dynamics for stable case ($\gamma^C = 12$).

6.3.3 Unstable scenarios and endogenous crashes

It has been demonstrated above, how the interplay of expectation formation of economic actors stochastic influences and the feedback between investment, consumption and cost dynamics generates endogenous business cycles in the model. Careful analysis allows to understand how the intensity of fluctuations is influenced by different important model parameters. One parameter with important implications on the intensity of the business cycles and the stability of

the economy is the intensity of choice parameter of consumers (γ^C), which governs the intensity of competition in the consumption goods market. A higher value of γ^C indicates a higher sensitivity of consumers to price differences between the firms, hence it is a measure of market competitiveness. In figure 8 we illustrate that if the intensity of competition is increased (i.e. γ^C is increased) firms price more aggresively and the investment growth in the upswing phases of the economy becomes more pronounced (Panel e). As can be seen this has a destabilizing effect on the economy and might lead to substantial crashes with recoveries. The overheating of the economy with associated cost driven price increases around month 300 can be clearly seen. Eventually the growth of (nominal) consumption budgets falls behind price increases inducing that consumption growth drops off, which leads to under-utilization of the largely expanded capacities (Panel b). Reinforcing feedbacks between unemployment increase, consumption decrease and cost increase lead to a sharp downturn in investment (Panel e), production and consumption, where also some firms have to leave the market due to bancrupcies (Panel f).

As is illustrated in Figure 9 these instabilities on the real side of the economy are strongly interlinked with the dynamics on the credit market. The overheating now appears in the form of a credit bubble that allows firms to finance their expansion of production capacity. Through the credit channel this leads to an overexposure of the banks to default risk.

In Figure 9 (Panel a and d) we see that if profits are negative the firm debt is increased, as external finance is now needed not only to pay for investments but also for interest and principle on old debts. During the period of expansion, as firms take on more risky financial positions, the firms' probability of default increases. As firms increase their leverage during the first 200 months, debt is increased, equity decreases, and the equity/asset ratio deteriorates (Panel c). The financial fragility of firms increases up to the point where they can no longer re-finance their debt and a sudden mass of insolvencies occurs. Each time there is a bankruptcy, the bankrupt firm writes down part of its outstanding debt and its equity/asset ratio improves (Panel c). If there are many bankruptcies at the same time this might cause a problem on the balance sheet of the bank. As a result, banks intermittently stop providing new loans altogether in this unstable setting. The number of new loans (Panel e) is much lower than in the stable case, however the volume of credit provided can be much higher (Panel d). Therefore the average amount of credit per loan is higher so also the risk to the banks due to the increased probability of failure on the part of the firms is much higher. From the bank data the build up of financial fragility can be seen in the bank's exposure (Panel f). Low values of exposure indicate the bank is not carrying a large risk. High values are correlated to the probability of default of the firms. As banks expand their balance sheet the total credit builds up, and the banks' exposure to default risk increases (Panel f). Total credit converges around a level much higher than in the stable base scenario. As the waves of bankruptcies start to accumulate the bank has to write-down bad debts (Panel b). This results in a sharp drop in total credit, and the bank's exposure is reduced (deleveraging). Several cycles of increased leveraging and subsequent deleveraging can be observed in the unstable scenario.

In summary, a higher intensity of competition on the consumption goods market yields an unstable scenario in which there are fewer firms, more financial fragility with a higher risk of default, banks having a higher exposure, and several episodes in which lending to firms comes to a complete standstill (Panel e).

Figure 10 illustrates the heterogeneity of firms in terms of the Herfindahl indices of output and prices, respectively. The plots indicate that in the unstable case, during a downturn there is a concentration effect when many firms are exiting the market and the heterogeneity in the firm data decreases.

6.3.4 Heterogeneity and firm size distribution

As pointed out several times the persistent heterogeneity of agents of the same type is an important feature of the Eurace@Unibi model. To conclude our brief documentation of simulation output generated by the model we illustrate this feature focusing on consumption good producers. Figure 11 shows log-log plots of the firm size distribution with respect to employees, capital stock and profits after 300 and after 400 months in the baseline scenario. For all three variables there is persistent heterogeneity, where the distributions exhibit fat tails and resemble Laplace distributions. So, also in this respect the simulation output is in accordance with empirical stylized facts. To illustrate the evolution of firm heterogeneity over time we depict in Figure 12 the dynamics of the output quantity of all 80 consumption good producers in a single run of the baseline scenario. The persistent heterogeneity, which actually seems to be increasing over time, can also be clearly seen in this figure. Furthermore, we can see that there is also considerable persistence with respect to the market share of each firm although there is also certain movement in this respect (e.g. the yellow firm initially performs very well, but then moves to the lower end of the size distribution and finally seems to disappear from the market).

Within the Eurace@Unibi framework the driving forces responsible for such features of the dynamics can be examined and understood by considering the different relevant time series (cost, sales, sales-expectation, etc.) on the individual agent and on the market level.

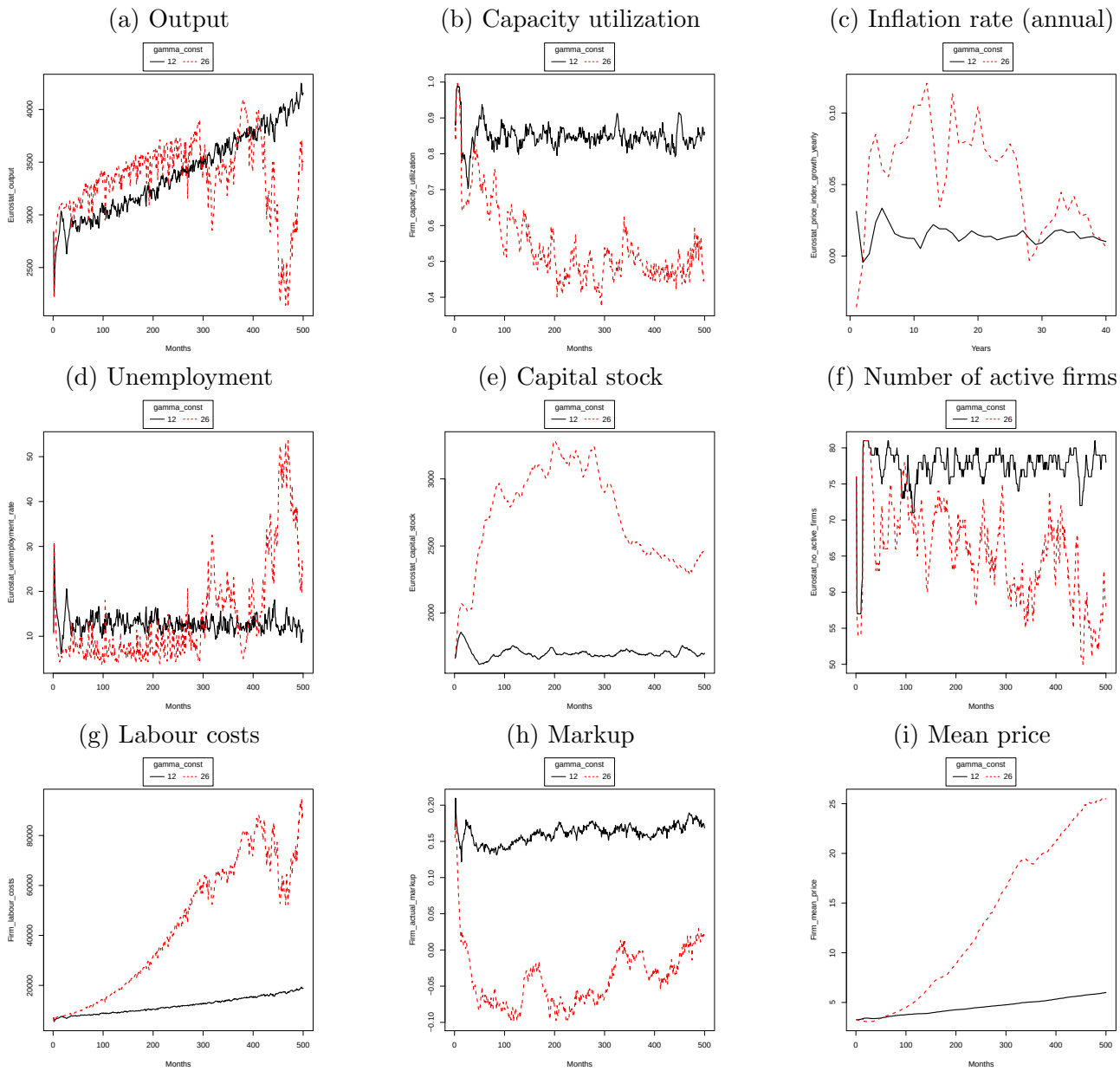


Figure 8: Macro variables in an unstable scenario with an endogenous crash ($\gamma^C = 26$).

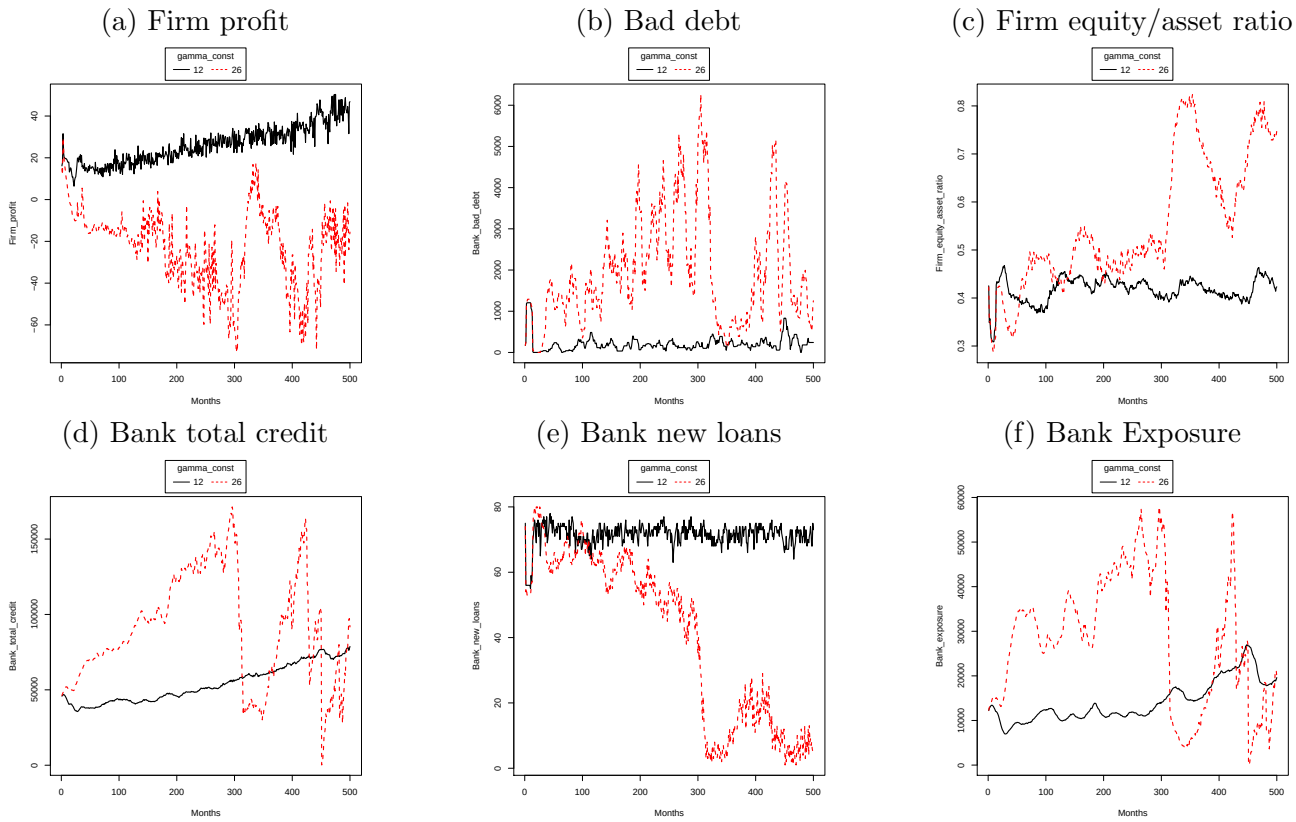


Figure 9: Dynamics of firm and bank data in an unstable scenario ($\gamma^C = 26$).

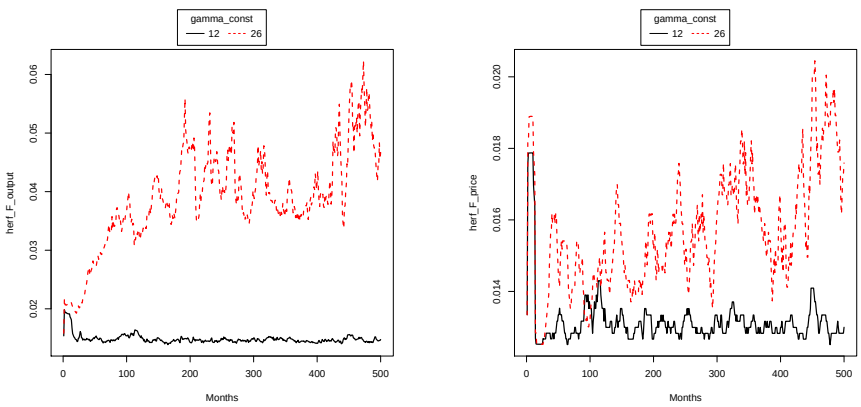


Figure 10: (a) Herfindahl index of firm output distribution, (b) Herfindahl index of firm price distribution.

(a) Firm no. employees (b) Firm capital stock (c) Firm profits

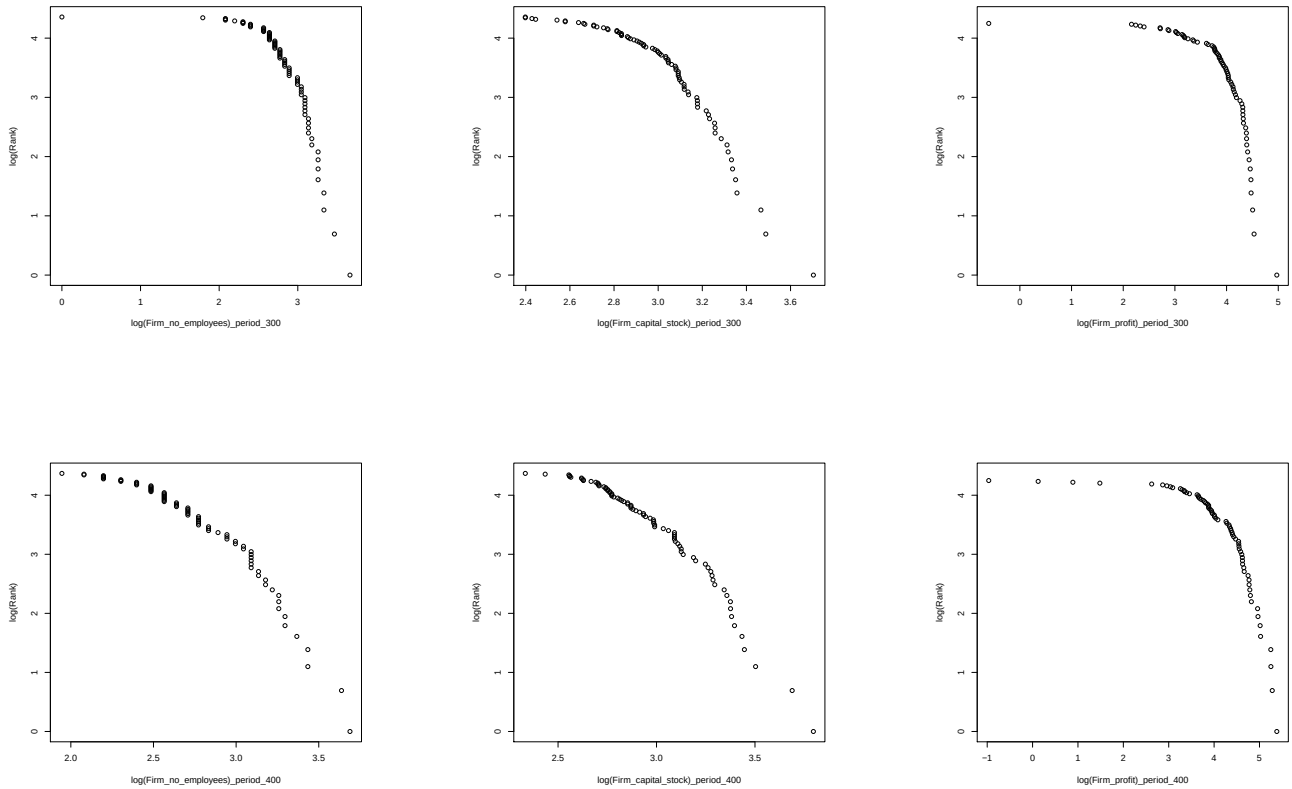


Figure 11: Distributions of firm size in terms of number of employees, capital stock and profits. Top Panel: distribution after 300 months, bottom panel: distribution after 400 months. For all three indicators of firms size, the support of the distributions become wider, indicating that the distribution becomes more skewed.

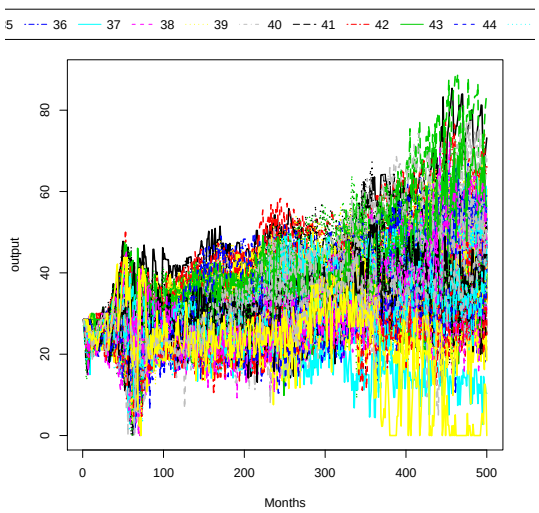


Figure 12: Dynamics of output quantities of all firms for a baseline run.

Appendix: Differences between the Eurace@Unibi and the original EURACE model

The Eurace@Unibi model is based on the model that was developed during the Eurace Project, but should not be identfied to it. It contains several extensions and simplifications with respect to the model at the end of the EURACE project in 2009. A list of features that differ is the following:

1. Consumption good producers determine their prices based on the estimated elasticity of their residual demand, where the estimation uses data from the test market. Hence firms' mark-up varies over time in response to changes in the economic environment. [In the Eurace model mark-up pricing with a fixed exogeneously given mark-up is used.]

2. Investment goods producers at each point in time produce and offer several vintages of capital that differ with respect to their quality and price, with newer vintages having a higher productivity than older ones. Investment good producers have decide how to price each of the vintages they are offering and consumption goods producers have to decide which vintage to select when investing in new physical capital. Decision procedures for each of these decsions have been added. [In the Eurace model there is just a single vintage of capital the productivity of which increases over time.]

3. To determine the output quantity of a consumption goods producers, the output of each employed vintage in the firm's capital stock is determined separately, under consideration of the skills of the firm's labor force, using a Leontief production function, and then aggregated. Thereby it is captured that if a firm employs some old vintages then the skills of the workers using these vintages are not fully used even if the average productivity of the firm's capital stock is above the workers skills. [In the Eurace model firm output is determined using a Cobb-Douglas production function and productivity is given by the minimum of average specific skills and average quality of the capital stock of the firm.]

4. Credit market: The banks pay a positive interest on deposits. The Central Bank pays the base rate on overnight deposits of the banks, and banks have to pay the base rate on their Central Bank debt (overdrafts on the standing facility). [In the Eurace model banks do not pay interest on deposits, nor does the Central Bank pay interest to banks.]

5. The financial market is highly stylized and simplified. There is a single risky asset that is a stock market index. Transactions follow from a rationing scheme, and the price mechanism is a cautious adjustment based on the excess demand for index shares. Dividends are paid on shares of the market index. Firms are not allowed to issue new shares, hence the number of outside shares remains constant. Asset allocation decisions of households are based on heuristic rules. [In the Eurace model there is a multi-asset market with shares of individual firms, banks and for government bonds. A centralized clearinghouse manages all trades by a clearing price mechanism. Firms are allowed to issue new shares to raise equity. Asset allocation decisions of firms are based on intertemporal optimization considerations using prospect theory utility functions.]

6. Balance sheets. The structure of the balance sheets has been completely overhauled and checked rigorously. A list of 33 rules for stock-flow consistency has been checked as a means of model verification (see Section 5).

7. Bankruptcy code. The code for firm bankruptcy has been extended to ensure that bankrupt firms are able to resurface (see Section 3.4).

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