Economic Growth
Chapter 4 : Growth and convergence in the Solow Model

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- Solow, Robert, **1956**, A Contribution to the Theory of Economic Growth, *Quarterly Journal of Economics*, 70, 65-94.
- Replying to Harrod's pessimistic conclusion
- $\rightarrow$ A very simple model to answer our fundamental question:
- "Why some countries are rich, and other are poor?"
- $\rightarrow$ A minimalistic model where the accumulation of the two main production factors (capital and labor) is the engine of growth

Purely based on the accumulation of factors, using some important simplifying neoclassical assumptions:
(**H1**) The country produces and consumes a single homogenous good (its level is Y);
(**H2**) Markets are perfectly competitive;
(**H3**) Production technology is exogenous and given;
(**H4**) It can be represented using a **neoclassical** production function, based on two substitutable inputs: capital (K), and labor (L);

$$Y = F(K, L) \tag{1}$$

A production function is neoclassical, if it fulfills three conditions:

1. Decreasing marginal productivities

$$\forall K > 0, L > 0, \quad \begin{array}{ll} \dfrac{\partial F}{\partial K} > 0, & \dfrac{\partial^2 F}{\partial K^2} < 0 \\ \dfrac{\partial F}{\partial L} > 0, & \dfrac{\partial^2 F}{\partial L^2} < 0. \end{array} \tag{2}$$

2. Constant returns to scale

$$F(\lambda K, \lambda L) = \lambda F(K, L), \quad \forall \lambda > 0. \tag{3}$$

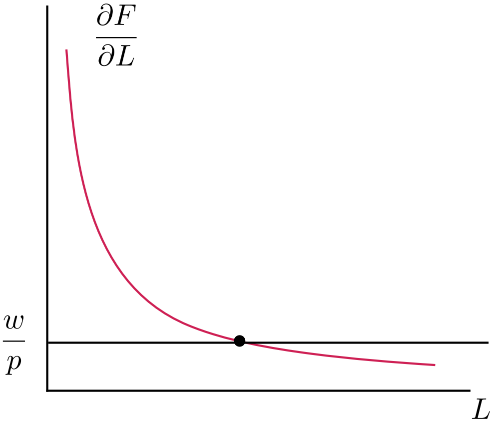
(F is homogenous of degree 1)

3. *Inada conditions* (Inada (1963))

$$\begin{array}{l} \lim_{K\to 0} F_K = \lim_{L\to 0} F_L = \infty, \\ \lim_{K\to\infty} F_K = \lim_{L\to\infty} F_L = 0. \end{array} \tag{4}$$

(the shape of F_K and F_L is hyperbolical)

Marginal productivities



- **Solow's trick:** Thanks to the constant returns to scale, the production function can be written in *per* capita *(intensive) form:*

$$Y = F(K, L) = L.F\left(\frac{1}{L}K, \frac{1}{L}L\right) = L.F\left(\frac{K}{L}, 1\right)$$

$$\Rightarrow y \equiv Y/L = F\left(\frac{K}{L}, 1\right) \equiv f(k) \tag{5}$$

with $k \equiv K/L$

- Using these new notations, we can redefine marginal productivities as:

$$\frac{\partial Y}{\partial K} = f'(k), \tag{6}$$

$$\frac{\partial Y}{\partial L} = f(k) - kf'(k).$$

- and the Inada conditions imply the following:

$$\lim_{k\to 0} f'(k) = \infty \text{ et } \lim_{k\to\infty} f'(k) = 0.$$

Other assumptions:

(H5) Aggregate consumption of households is given by a Keynesian consumption function:

$$C = c.Y \Rightarrow S = (1 - c)\,Y = s \cdot Y \qquad (7)$$

(H6) Labour participation rate of households is constant, and the active population, and hence the labour supply of households (L) increases with the same speed as the population, the instantaneous rate n:

$$\frac{d\log(L)}{dt} = \frac{dL/dt}{L} = \frac{\dot{L}}{L} = n \qquad (8)$$

- In this course, we will simplify even more the production sphere, and assume a Cobb-Douglas function:

$$Y = F(K, L) = K^{\alpha} L^{(1-\alpha)}, \quad \alpha \in [0, 1]. \tag{9}$$

- Returns to scale are constant $(\alpha + (1-\alpha) = 1)$:

$$F(\lambda K, \lambda L) = (\lambda K)^{\alpha} (\lambda L)^{(1-\alpha)} = \lambda F(K, L) \tag{10}$$

- Moreover, the price of the product is normalized to unity: $p = 1$.
- In perfect competition, the firms are price–taker, and they maximize their profits given the prices of the product and all inputs:

$$\max_{K,L} F(K, L) - rK - wL$$

where r is real interest rate in the economy, and w, the real wage.

- Profit maximization $\rightarrow$

$$w = \frac{\partial F}{\partial L} = (1-\alpha)\frac{Y}{L} \quad (11)$$

$$r = \frac{\partial F}{\partial K} = \alpha\frac{Y}{K} \quad (12)$$

- Homogeneity of the production function, constant returns to scale and the Euler identity imply that

$$wL + rK = Y$$

  The output is exhausted once we pay all factors at their marginal productivities (see equations (11)–(12)).
- This technology, with possibility of substitution, and decreasing marginal productivities constitutes the main difference of this model with Harrod's keynesion model.
- Solow shows that the instability of the growth on the "edge of the knife" is essentially due to the absence of substitution between capital and labor in the Harrod's model.

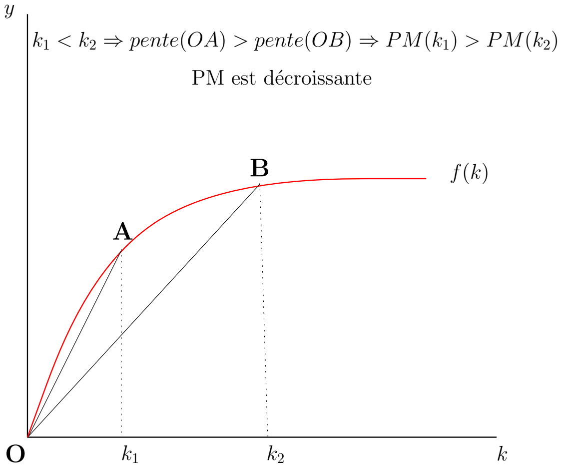
- Stylized facts on growth $\leftrightarrow$ *per capita* production (y).
- We can reformulate the model in these terms:

$$k \equiv \frac{K}{L} \quad (\text{avec } \frac{L}{L} = 1)$$

$$y \equiv \frac{Y}{L} = f(k) = \frac{F(K, L)}{L} = \frac{K^{\alpha} L^{(1-\alpha)}}{L} = \left(\frac{K}{L}\right)^{\alpha}$$

$$y = f(k) = k^{\alpha} \tag{13}$$

Per capita production function of the Solow model



→ Decreasing returns

Dynamics of the capital

- **First fundamental equation** of the Solow model: Equation 13.

$$y = f(k) = k^{\alpha}$$

- Second fundamental equation of the model corresponds to capital accumulation (and hence the dynamics).
- Variation of capital stock = Investment - Depreciation of the capital :

$$\dot{K} \equiv \frac{dK}{dt} = I - \delta K \tag{14}$$

where δ is the depreciation rate of the physical capital, and it is constant.

Investment

We have a closed economy
→ Investment = Savings (good market equilibrium):

$$I = S = (1 - c) \cdot Y = s \cdot Y \quad (15)$$

$$\dot{K} = sY - \delta K \quad (16)$$

Dynamics of per capita physical capital

Variation in time of per capita capital:

$$k = \frac{K}{L} \Rightarrow \log(k) = \log(K) - \log(L)$$
$$\Rightarrow \frac{d\log(k)}{dt} = \frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} = \frac{sY - \delta K}{K} - \frac{\dot{L}}{L} \quad (17)$$

Equation 8 + equilibrium of the labor market → growth rate of the labor factor used by this economy:

$$\frac{\dot{L}}{L} = n \Rightarrow L(t) = L_0 e^{nt} \quad (18)$$

Combining all these results→ Equation 17 becomes:

$$\frac{\dot{k}}{k} = \frac{sY}{K} - \delta - n = \frac{sy}{k} - \delta - n.$$

$$(y/k = (Y/L)/(K/L) = Y/K)$$

The second fundamental equation of the Solow model.
Combining both fundamental equations →
Fundamental equation of the model on the dynamics of the per capita physical capital:

$$\boxed{\dot{k} = s \cdot f(k) - (\delta + n) \cdot k} \qquad (19)$$

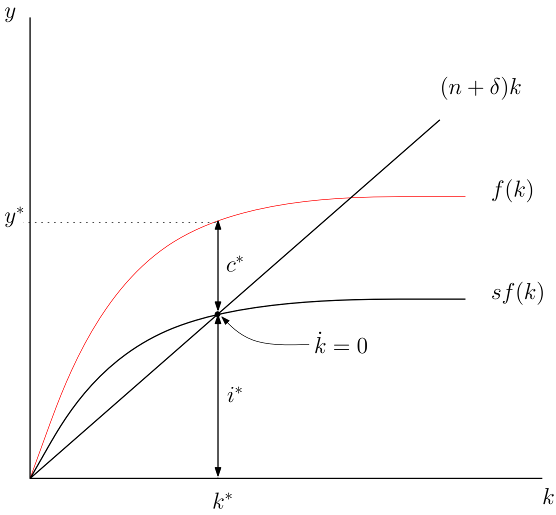
- Dynamics of the model are driven by its two fundamental equations: (13) et (19)
- If the economy starts at an initial state with $k_0 = K_0/L_0$,
- the first fundamental equation $\rightarrow$ production level at each instant $\rightarrow$ saving and investment,
- the second fundamental equation $\rightarrow$ how these elements determine the accumulation of the (per capita) capital
- $\rightarrow$ to unfold the dynamics of the economy in time.

$$
\begin{array}{ccccc}
\boxed{k_0 = \frac{K_0}{L_0}} & \xrightarrow{f(k)} & y & \xrightarrow{s,\delta} & \dot{K} \\
n\downarrow & & & & \downarrow \\
L_1 & \longrightarrow & \boxed{k_1} & \longleftarrow & K_1
\end{array}
$$

The solow diagram conveniently represent these mechanisms as a graphic.

- Solow diagram$\rightarrow$ summaries all variables about this economy, as functions of the per capita capital
- Dynamics of the capital $\leftarrow$ Balance between two forces:
    - Accumulation of the capital (thanks to investment) $= sf(k)$;
    - Social depreciation of capital $= (n+\delta)k$.
- The growth rate of $k \leftarrow$ the gap between the curves corresponding to these two forces: $sf(k)$ et $(n+\delta)k$

The Solow diagram



- At the intersection of these two curves, we have:

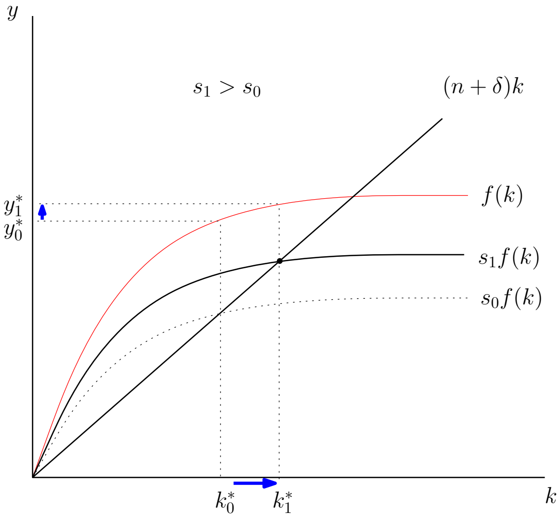
$$\frac{\dot{k}}{k} = 0 \quad \Rightarrow \dot{k} = 0, \quad k = k^*$$

- This is the only balanced growth path **(BGP)** of this economy, if we exclude the dead state ($k^* = 0$).
- We have on this BGP: $k = k^* = Cste \Rightarrow \dot{k} = 0$ and hence, $\gamma_k = \dot{k}/k = 0$: a steady state **(SS)**.
- Out of this steady state, we have either an intensification of the capital in the economy $\left(\dot{k} > 0\right)$, or a dispersion of the capital $\left(\dot{k} < 0\right)$.

- Comparative statics in this model $\rightarrow$ observing the evolution of k from a steady state, after a shock on the economy
  1. Reaction of the equilibrium of the economy to the shock
  2. Transition towards the new equilibrium (stability?)
- We will consider two structural shocks on this simple economy:
  - an increase of the saving rate;
  - a stronger demographic growth.

An increase of the saving rate

- Starting from an SS corresponding to $s = s_0$
- an increase of s: $s = s_1 > s_0$
- Effect on the BGP? And on k^* and y^*?



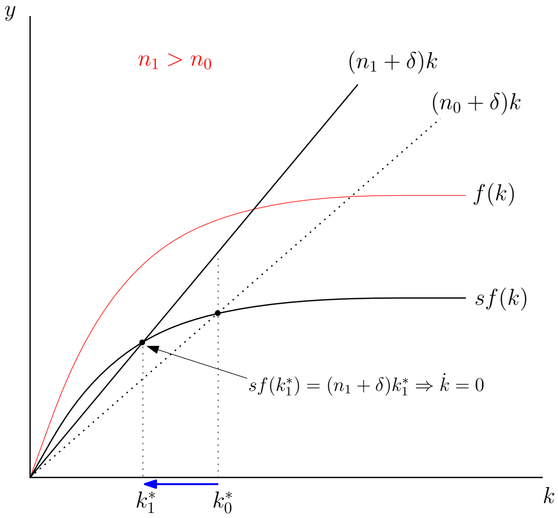
$$s_0 \rightarrow s_1 > s_0 \Rightarrow k_1^* > k_0^* \text{ et } y_1^* > y_0^*$$

A stronger demographic growth

- Lets start again from the initial SS
- and consider an increase of the demographic rate:

$$n : n_0 \to n_1 > n_0$$

- • Effect on the BGP? And on k^* and y^*?



$$n_0 \rightarrow n_1 > n_0 \Rightarrow k_1^* < k_0^* \text{ et } y_1^* < y_0^*$$

Etat stationnaire

- Per capita capital on the SS:

$$\dot{k} = sk^{\alpha} - (n+\delta)\,k = 0$$

$$k^* = \left(\frac{s}{n+\delta}\right)^{1/(1-\alpha)} \tag{20}$$

- Per capita production on the SS:

$$y^* = f\left(k^*\right) = \left(\frac{s}{n+\delta}\right)^{\alpha/(1-\alpha)} \tag{21}$$

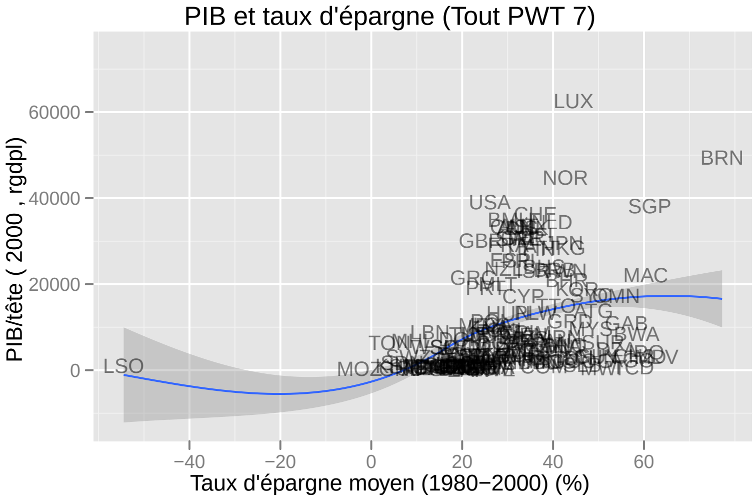
- Equation (21) → A first answer by this model to the fundamental question: "Why some countries are rich, and other are poor?"

Proposition

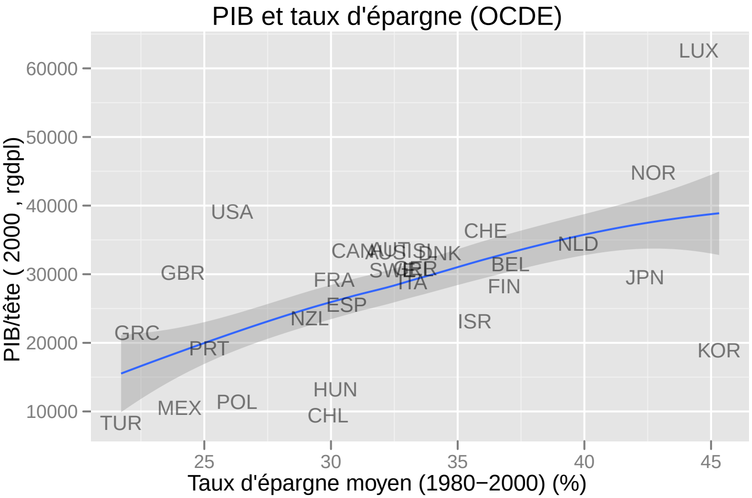
Countries that have a higher saving/investment rate have a tendency of being "richer", and the ones who have a stronger demographic rate, "poorer".

- Confrontation with empirical observations?

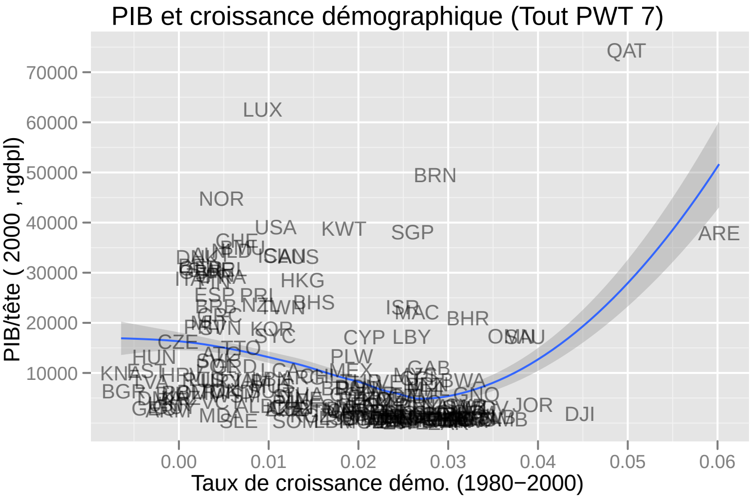
Investment rate and growth: All PWT 7



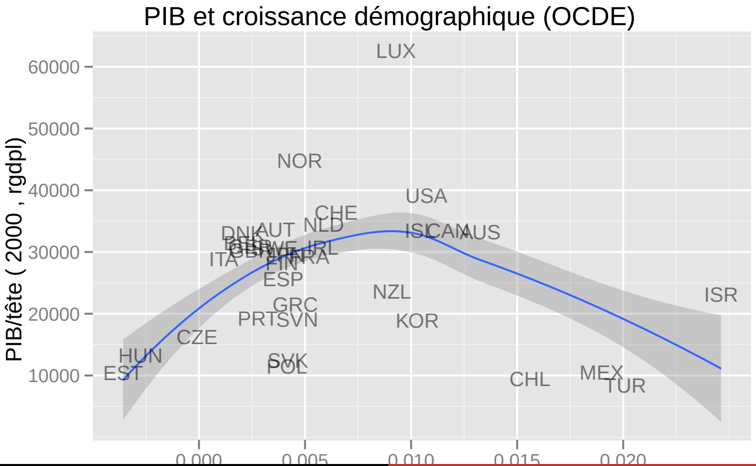
Investment et growth: OCDE countries



Demographic growth rate and economic growth: All PWT 7



Demographic growth rate and economic growth: OCDE countries



Golden rule of consumption (and welfare)?

- A higher saving rate yields a richer BGP.
- But, the highest level of saving ($s = 1$) is also the level at which the consumption is zero!
- and $s = 0$ yields a dead economy, without any possibility of consumption
- Can we find an intermediate level of saving rate that maximizes consumption, and hence welfare, in this simple economy?
- We can check this by observing how the consumption level on the BGP varies with the saving rate.

Consumption levels on the BGP

- On the BGP, for a given level of n, δ, we have the corresponding level of the capital, which increases with s

$$k^* (s), \ \frac{dk^* (s)}{ds} > 0 \tag{22}$$

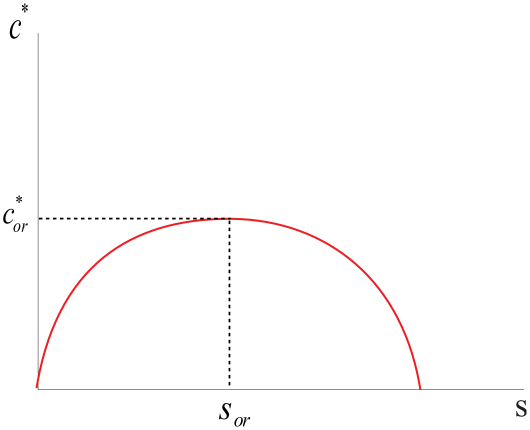
$$\text{and } SS \Rightarrow s \cdot f(k^*(s)) = (n+\delta) k^*(s) \tag{23}$$

$$\text{Since } \quad c^*(s) = (1-s) \cdot f(k^*(s))$$

$$c^*(s) = f(k^*(s)) - (n+\delta) k^*(s) \tag{24}$$

The latter is the consumption level on the BGP, and it depends on s through its impact on k^*.

Saving rate and consumption on the BGP



There must be an optimal rate of saving in terms of consumption and welfare!

Golden rule on the optimal saving rate of the economy

- Let's call s_{or} this optimal rate of saving:

$$s_{or} = \arg\max c^{*}(s),$$

$$\frac{dc^{*}}{ds} = \left[f'(k^{*}) - (n+\delta)\right] \cdot \frac{dk^{*}}{ds} = 0$$

$$\Rightarrow \boxed{f'(k_{or}) = (n+\delta)} \tag{25}$$

$$\text{with } k_{or} = k^{*}(s_{or})$$

$$c_{or} = f(k_{or}) - (n+\delta) \cdot k_{or} \tag{26}$$

- With this saving rate, the per capita marginal productivity must exactly compensate the social depreciation rate of the economy.
- With the Cobb--Douglas production function, this condition implies that

$$s_{or} = \alpha \tag{27}$$

- The BGP of the basic model = a SS → per capita variables are constant on the BGP.
- But, the level variables (Y, S, C, K) grow with the same speed as the population

$$\frac{\dot{k}}{k} = \frac{\dot{y}}{y} = 0 \Rightarrow \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{L}}{L} = n$$

since

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} = \frac{\dot{K}}{K} - n = 0$$
$$\Rightarrow \frac{\dot{K}}{K} = n$$

And the stylized facts observed in the introduction?

The model is able to exhibit at the long term (on the BGP):

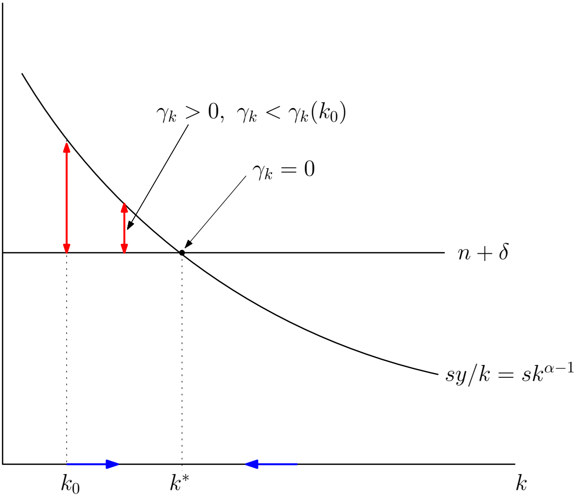
- Variation of the per capita GDP between countries with different structures (s, δ, n, α);
- Kaldor Fact: Constant Capital–Output ratio (K/Y) (since k and y are constants);
- A constant k yields a constant return on the capital (=real interest rate = marginal productivity of k^* on the BGP)
- But...

- A very important stylized fact is missing: the sustained growth of the per capita GDP (y) !
- In this model $\rightarrow$ any economy can grow in the short term, but its per capita GDP cannot grow in the long term
- If a country gets temporarily out of its SS (following a shock, for example),
- $\rightarrow$ it will follow a transition path, and
- $\rightarrow$ land on a new SS, corresponding to the new economic conditions.
- The closer the economy to the SS, the slower the growth process..
- Why do we have this result?

- The culprit: decreasing returns on capital
- The average productivity of capital (AP) is decreasing ($\alpha < 1$) in the fundamental dynamic equation of the model

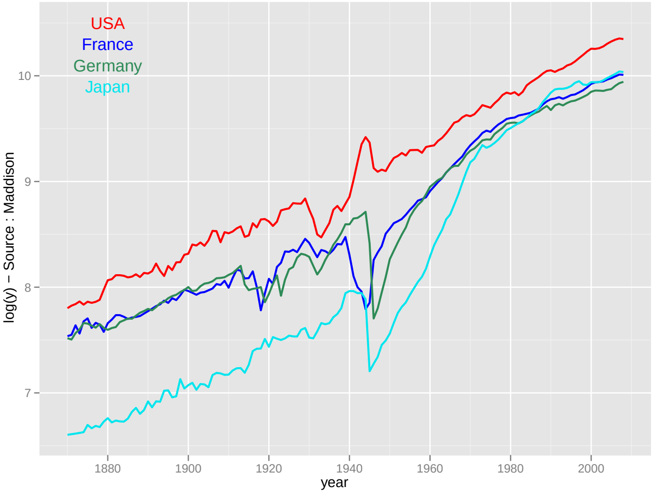
$$\gamma_k = \frac{\dot{k}}{k} = sk^{\alpha-1} - (n+\delta) = s \cdot \frac{f(k)}{k} - (n+\delta) \qquad (28)$$

- $\Rightarrow$ when k increases $\rightarrow$ the growth rate of k decreases (because $PM_k = f(k)/k$ decreases).
- γ_y is proportional to $\gamma_k \rightarrow \gamma_y$ also decreases.
- We can represent these forces on a figure.


$\gamma_k > 0,\ \gamma_k < \gamma_k(k_0)$
$\gamma_k = 0$
$n + \delta$
$sy/k = sk^{\alpha-1}$
k_0
k^*
k

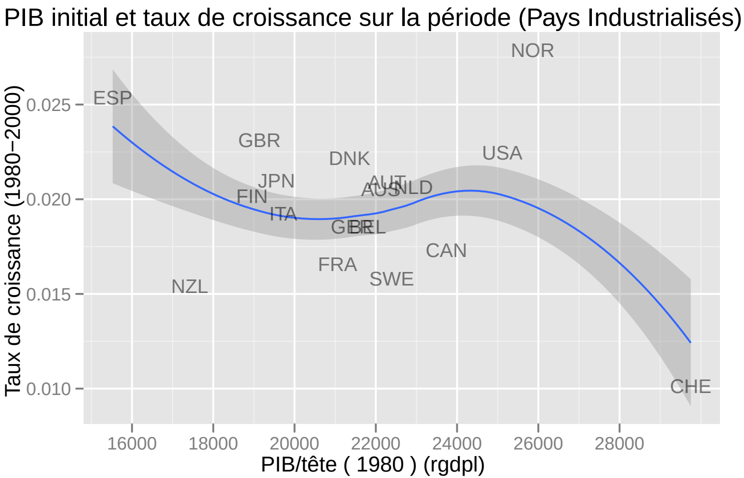
- The growth process becomes necessarily exhausted in per capita terms.
- This definitely contradicts the sustained growth period we observe since the Industrial Revolution!
- This simple model seems to definitely neglect some other engines of growth.
- That we will introduce later.

Baumol: Convergence between industrialized countries ?

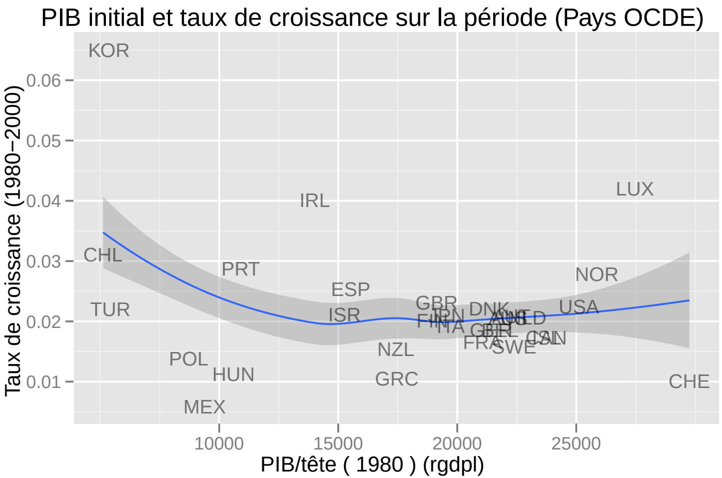


- Catching–up only if the lagging countries can run faster than the leaders
- The poorer a country, the higher its growth rate
- $\rightarrow$ Convergence between the income levels of different countries (**absolute convergence**)
- **Question:** Do we observe such a convergence?

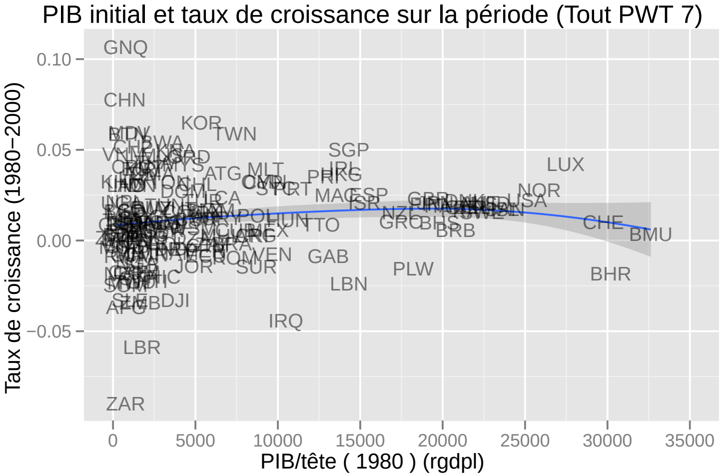
Convergence between industrial countries (PWT) ?



Quid of OECD countries?



And in a larger set of countries? ?



- Absolute convergence is not a global phenomenon!
- What is the teaching of the solow model on the convergence process?

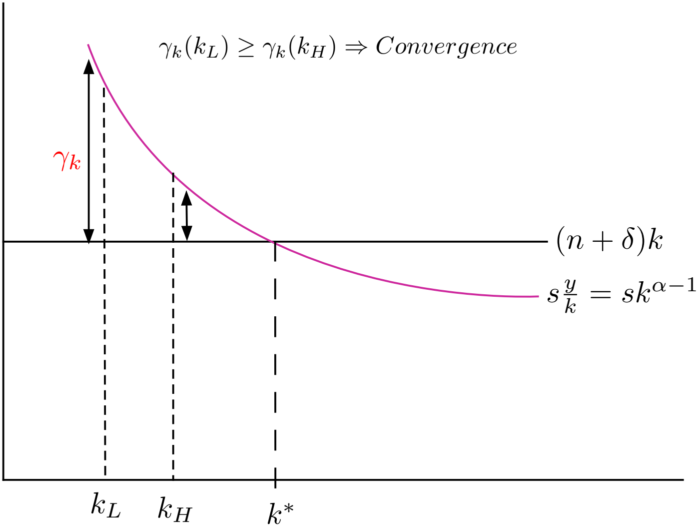
Vectors of convergence in the Solow model?

The convergence process is driven by the growth rates, and we have seen that they are governed in this model b:

$$\frac{\dot{k}}{k} = s\frac{y}{k} - (n + \delta)$$

with

$$y/k = k^{\alpha-1}$$


$\gamma_k(k_L) \geq \gamma_k(k_H) \Rightarrow Convergence$
γ_k
$(n+\delta)k$
$s\frac{y}{k} = sk^{\alpha-1}$
k_L
k_H
k^*

Model's main result: Only conditional convergence

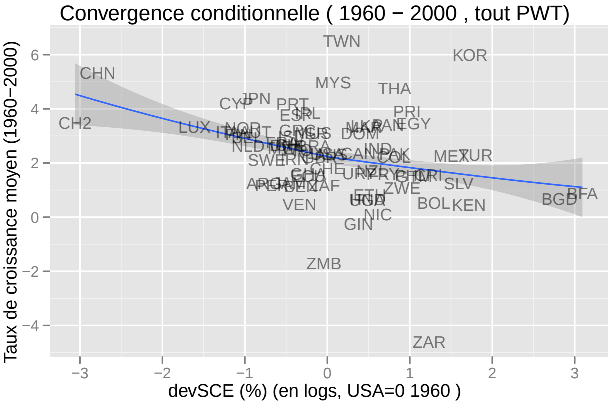
Proposition

Convergence would only take place between economies with similar SS (hence, similar structures): Absolute convergence is not possible between countries with different structures.

Proposition

Only convergence conditioned by economic structures is possible in this model: The more a country far from its SS, the stronger, the growth rate.

Conditional convergence?



The correlation between the initial distance to the SS at 1960, and the average growth rate during 1960--2000 is -0.335 < 0, and its is significantly different from zero ($p - value = 0.003767$).

Econometric tests of conditional convergence

- Do poorer countries benefit from higher rates of growth ?
- If we control for their structural disparity : Estimating

$$\frac{1}{T} log \frac{y_{t+T}}{y_t} = \beta_0 + \beta_1 log y_t + \beta_2 X_t + u_t$$

where X_t is a vector of structural variables (like n, s, δ), controlling for the steady state of each country.

- We conclude for conditional convergence if the estimated value for $\beta_1 < 0$: lower growth rate results from the distance to the BGP.

Factors of convergence ?

- Sala-I-Martin, Xavier X. 1997. I Just Ran Two Million Regressions. *The American Economic Review*, 87(2), 178–183. (XSM)
- Trying to conclude on the huge empirical literature on convergence
- mobilizing a different set of explanatory variables in each study (the X_t vector above).
- 62 variables collected by XSM, each found significative at least in one regression.
- **Question :** which variables are really correlated with growth ?

Econometric model

- Trying to estimate the distribution of the estimator of each coefficient
- Always including three variables : y_{1960}, life expectancy in 1960, primary-school enrollment rate in 1960,
- combining each potentially important variable with these three + with sets of three variables selected from the remaining 58 : 30856 regressions per variable = 2 million regressions in total.

Results ?

- **Regional variables :** Sub-Saharan Africa, Latin America and former Spanish colonies (all negatively correlated), and absolute latitude (better to be far from the equator)
- **Political variables :** Rule of law and civil liberties (positive), number of revolutions and military coups, and war (negative)
- **Religious variables :** Confucian, Buddhist, and Muslim (positively), Protestant and Catholic (negative)
- **Types of investment :** Equipment and non-equipment (positively, but stronger effect for the equipment)
- **Sectoral weights :** Fraction of primary in exports, and Fraction of GDP in mining (both positive)
- **Openness :** Number of years the economy has been open between 1950 and 1990 (positive)
- **Economic system :** Degree of capitalism (positive)

Is there something else missing from the Solow model ?

- Which engines of growth are incorporated in the basic Solow model ?
  - Demographic growth (increase of the labour supply) ;
  - Accumulation of capital.
- This model shows that these engines are not enough for securing a persistent growth of GDP/worker.
- Have we neglected some other important engine ?

- We assumed that the economy uses the same technology all along !
- But technical progress has been very persistent since the Industrial Revolution !
- We really should take this stylized fact into account.

- Consider an augmented production function :

$$Y = F(K, AL) = K^{\alpha} \cdot (AL)^{1-\alpha} \tag{29}$$

- **New element :** A represents the technological level of the economy
- Technical progress $\leftrightarrow$ increase of A in time $\leftrightarrow$ each unit of labor becoming more and more productive.
- $\rightarrow$ "labor augmenting" technical progress, "neutral" in the terminology of Harrod

- Extension of Solow's basic model :
- Solow, R., 1957, "Technical Change and the Aggregate Production Function", *Review of Economics and Statistics*, 39, 312-320.
- Incorporation of *exogenous technical progress*
- $\rightarrow$ Continuous increase of the technological level, A, at a constant rate :

$$\frac{\dot{A}}{A} = g \Leftrightarrow A = A_0 \cdot e^{gt} \tag{30}$$

- "*Manna from Heaven*"

- Other mechanisms of the model do not change.
- As the accumulation of the capital :

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta \tag{31}$$

- Production function is now given by :

$$y = \frac{Y}{L} = \frac{K^{\alpha} \cdot (AL)^{1-\alpha}}{L}$$
$$y = \left(\frac{K}{L}\right)^{\alpha} A^{1-\alpha} \left(\frac{L}{L}\right)^{1-\alpha} = k^{\alpha} A^{1-\alpha} \tag{32}$$

- And growth ?
- Logarithmic derivative of the production function $y = k^\alpha A^{1-\alpha}$:

$$\gamma_y = \frac{\dot{y}}{y} = \alpha \frac{\dot{k}}{k} + (1-\alpha)\frac{\dot{A}}{A} = \alpha \cdot \gamma_k + (1-\alpha)\cdot g$$

- And, equation (31) gives us

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta \tag{33}$$

- $\rightarrow$ On the BGP, the growth rate of K would be constant if and only if Y/K is constant (because s, δ are constant)
- $\rightarrow \gamma_Y = \gamma_K \Rightarrow \gamma_y = \gamma_k \Rightarrow$

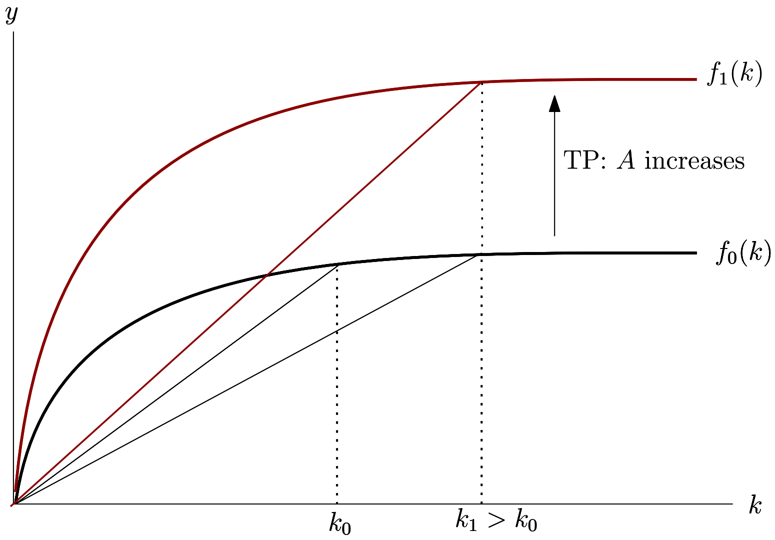
$$\gamma_y = \alpha \cdot \gamma_y + (1-\alpha)\cdot g \Leftrightarrow \gamma_y = \gamma_k = \gamma_A = g \geq 0 \tag{34}$$

- Now, with this exogenous technical progress,
- $\rightarrow$ Per capita Capital, consumption, but also GDP, grow at a constant and positive rate, g.
- While the level variables grow at a higher rate

$$\gamma_Y = \gamma_K = \gamma_C = n + g$$

- Technical progress continuously increases the productive capacity of the economy
- and compensates the decreasing returns

Technical progress and average productivity



- Value of y on the BGP :

$$y_t^* = \left(\frac{s}{\delta + g + n}\right)^{\alpha/(1-\alpha)} A_t \tag{35}$$

- y grows at the same rate as A !

$$\gamma_y = \gamma_A = g$$

- Solow, R., 1957, "Technical Change and the Aggregate Production Function", *Review of Economics and Statistics*, 39, 312-320.
- $\rightarrow$ Evaluating empirically the factors driving the growth processes in different countries and periods
- Including potential technical progress
- $\rightarrow$ Growth accounting

- Solow starts from the following productions function :

$$Y = BK^{\alpha}L^{1-\alpha},$$

- Increase of B $\rightarrow$ Technical progress (TP)
- Growth rate of Y :

$$\gamma_Y = \frac{\dot{Y}}{Y} = \frac{\dot{B}}{B} + \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L} \tag{36}$$

- We hence take into account now that B can change in time.
- $\rightarrow$ TP $\rightarrow$ growth of the *total productivity* of factors.
- $\rightarrow$ Both factors are influenced by the TP.

- Growth of Y = weighted average of the growth of K, L and B.
- We can consequently measure the contribution of each factor.
- $\rightarrow$ "Growth accounting" !
- Empirical study : USA between 1960 et 1990 (source Jones (1998)).

Period	$\dot{Y}/Y$	$\dot{y}/y$
1960 – 70	4.0	2.2
1970 – 80	2.7	**0.4**
1980 – 90	2.6	1.5
1960 – 90	3.1	1.4

- Clear slowing of the GDP/worker during the decade 1970-80.
- How can we explain this ? What is the culprit ? Weaker accumulation of the factors ?

- Empirical studies estimates in general $\alpha = 1/3$, which would give :

$$\gamma_Y = \frac{1}{3}\gamma_K + \frac{2}{3}n + \gamma_B$$

- Contribution to growth of the factor capital : $\frac{1}{3}\gamma_K$
- • Contribution to growth of the factor labor : $\frac{2}{3}n$
- We can know in this equation the values of γ_K, n and γ_Y
- We can compute the residual $\rightarrow \gamma_B$

$$\gamma_B = \gamma_Y - \left(\frac{1}{3}\gamma_K + \frac{2}{3}n\right)$$

- We can consequently decompose the growth rate of the Y and evaluate the contribution of each factor.

On the period 1960–90 :

		Contributions de			
Période	$\dot{Y}/Y$	K	L	PTF	$\dot{y}/y$
1960 – 90	3.1	0.8	1.2	1.1	1.4

- an average growth rate of 3.1% for Y
- 0.8 of these 3.1 points $\leftarrow$ accumulation of capital $\left(\frac{1}{3} \cdot \dot{K}/K = 0.8\%\right)$,
- 1.2 points $\leftarrow$ growth of the labor force $\left(\frac{2}{3} \cdot n = 1.2\%\right)$
- the remaining 1.1 points cannot be explained by the evolution of these factors
- $\leftarrow$ Evolution of the total productivity (TP, $B = A^{1-\alpha}$, $\dot{B}/B = 1.1\%$).

		Contributions de			
Période	$\dot{Y}/Y$	K	L	PTF	$\dot{y}/y$
1960 – 70	4.0	0.8	1.2	1.9	2.2
1970 – 80	2.7	**0.9**	**1.5**	**0.2**	0.4
1980 – 90	2.6	0.8	0.7	1.0	1.5
1960 – 90	3.1	0.8	1.2	1.1	1.4

- We observe for 70-80 that the accumulation of factors has not slowed, but the growth of y has slowed
- $\rightarrow$"Productivity paradox"
- Explanation : the slow growth can be explained by the slowing of the total factor productivity (hence, of the TP).

The jury is out on the reasons for which this slowing has taken place in that decade :

1. Increase of the energy cost, following the Oil shock ;
    - (but the price of the energy has decreased in the following decade, without yielding a growth similar to the period before the Oil shock)
2. Modification of the sectoral allocation of labour, the share of the services has increased, and this sector has lower productivity than the manufacturing.
    - (Possible because the productivity has gained back in the 80's its previous level in manufacturing, without yielding a comparable global growth level)
3. Reduction of the R&D expenses in the 60s.

- Many other applications of this accounting following Solow's initial study
- Alwyn Young : the Asian tigers (initial high growth mainly explained by the accumulation of the factors).
- Difficulty of
  - disentangling of TFP from the accumulation of the capital,
  - counting for qualitative dimensions of the TFP (better products)
  - taking into account systematic over-estimation of capital accumulation in many countries (Hsieh argues that Young's conclusion is due to this effect)
  - γ_B catches everything but capital and labor

- What is the origin of technical progress ?
- Exogenous ? Manna from heaven ?
- Or endogenously generated by the economic activities of economic agents ?
- Understanding the creation of new ideas and technologies.
- $\rightarrow$ Course with Peter Howitt

Are we missing something else ?

- Quantitative increase of the labour force
- but we have neglected its qualitative evolution
- Tremendous investment of capitalist countries in general and specialized education
- Could this play a role in their growth process ?