

Economic Growth

Chapter 2 : Fragility of growth in a simple Keynesian model

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- Can we extend the static Keynesian framework to analyze the capitalist growth process?
- Including in this static context the dynamics of labour supply (demographic growth), and of capital (investment).
- If yes, what can we learn about the economic growth in such a context?
- Harrod, R. F. 1939. *An Essay in Dynamic Theory*. *The Economic Journal*, 49(193), 14.
- Domar, E. D. 1947. Expansion and employment. *The American Economic Review*, 34–55.
- Harrod, R. F. 1960. Second Essay in Dynamic Theory. *The Economic Journal*, 70(278), 277.

“Once the mind is accustomed to thinking in terms of trends of increase, the old static formulation of problems seems stale, flat and unprofitable.”

- A very simple model, of a country producing only one good, whose production, at each instant, is Y_t .
- The production capacity of the country depends on the dynamics of the capital (investment), and of labour supply (demographic growth).
- As such, two independent dynamic processes.
- Ability of a capitalist economy to harness these two horses to drive a balanced growth path ?
- To establish and preserve the equilibria on both the good and the labour markets?
- Hence, to solve two problems:
 - Short term problem: Existence of equilibrium
 - Long term problem: Stability of the equilibrium

A simple framework

- A closed economy.
- One commodity, produced using capital and labour
- Capital stock increases thanks to investment
- Labour supply increases following the demographic growth at an instantaneous growth rate of n
- Two markets: The good market and the labour market
- Balanced growth: Necessity of establishing equilibria on both markets

Production of the good

- It is obtained by combining a fixed proportion of capital and labour.
- The coefficient of capital is constant at the equilibrium: μ ,
- and given a capital stock, if there is enough labour, the economy can produce an output of

$$Y_t = \frac{1}{\mu} K_t \text{ ou } K_t = \mu Y_t \quad (1)$$

Dynamics of the capital stock

- The capital stock can increase (net of depreciation) as a result of the investment effort of the economy:

$$\frac{dK}{dt} = \dot{K}_t = I_t \quad (2)$$

- In this closed economy, the equilibrium of the good market is possible *iff* the investment absorbs all saving:

$$\dot{K}_t = I_t = S_t \quad (3)$$

- Investment results from the expectations of the firms on the growth of the demand, \dot{Y}_t (the accelerator),
- and households save a fraction $s = 1 - c$ of their real income (Y_t) (the Keynesian multiplier).
- The resulting equilibrium conditions hence is:

$$I_t = \mu \dot{Y}_t = S_t = sY_t \quad (4)$$

Harrod's first problem

- Under these conditions, the growth rate of the economy that would warrant the equilibrium of the good market becomes:

$$\boxed{\frac{\dot{Y}_t}{Y_t} = \frac{s}{\mu} \equiv \gamma_w} \quad (5)$$

- This growth rate, and hence the good market equilibrium can only persist iff the expectations of the firms, and the consumption behavior of the households remain compatible in time.

"I use the unprofessional term warranted instead of equilibrium, or moving equilibrium, because, although every point on the path of output is an equilibrium point in the sense that producers, if they remain on it, will be satisfied, and be induced to keep the same rate of growth in being, the equilibrium is, for reasons to be explained, a highly unstable one."

- In this case, the production of the economy should evolve along the following trajectory:

$$Y_t = Y_0^K \cdot e^{\gamma_w t} \text{ with } Y_0^K = \frac{1}{\mu} K_0. \quad (6)$$

- Otherwise, the growth rate of the economy may be lower, and disequilibrium may appear (rationing on the good market).

Some complications to this simple story...

- Harrod considers several questions that could complicate these relations:
 - The potential difference between ex ante investment, and ex post investment (the former may be higher than the latter ($= S$) \rightarrow Expansion of the system (a Keynesian idea).
 - The time lag between the investment and the resulting production (neglected by Harrod for being small on the BGP).
 - Potential difference between ex ante and ex post saving (neglected again).
 - Relationship between the static equilibrium and the dynamic one: if ever $\gamma > \gamma_w \Rightarrow \gamma \nearrow$.

“Thus in the dynamic field we have a condition opposite to that which holds in the static field. A departure from equilibrium, instead of being self-righting, will be self-aggravating. γ_w represents a moving equilibrium, but a highly unstable one.”

But, we should not forget the other production factor: the labor supply

Equilibrium under full participation rate

- the equilibrium condition is given by:

$$\begin{aligned} \text{Labour demand} &= \text{Labour supply} \\ L_t &= N_t \end{aligned} \tag{7}$$

- At the instantaneous equilibrium of the economy, the labor coefficient is also constant: λ ,
- and the labor demand corresponding to the production level Y_t is simple given by:

$$L_t = \lambda Y_t \tag{8}$$

- Labor market equilibrium condition:

$$L_t = \lambda Y_t = N_t \Leftrightarrow Y_t = \frac{1}{\lambda} N_t \tag{9}$$

Natural growth rate

- and the (*natural*) growth rate that would be compatible with full employment is:

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{N}_t}{N_t} + \frac{(1/\lambda)}{(1/\lambda)} \equiv \gamma_n$$

$$\gamma_n = n + \ell \quad (10)$$

where ℓ is the growth rate of labor productivity (if any).

- The trajectory of the economy compatible with full employment would be:

$$Y_t = Y_0^L \cdot e^{\gamma_n t} \quad \text{with } Y_0^L = \frac{1}{\lambda} N_0. \quad (11)$$

- Growth rate is compatible with the good market equilibrium if:
 $\gamma_w = s/\mu$
- Growth rate is compatible with the labor market equilibrium if:
 $\gamma_n = n + \ell$.
- Starting from an equilibrium, the growth rate can only preserve it, iff

$$\gamma = \gamma_w = \gamma_n \Leftrightarrow \boxed{\frac{s}{\mu} = n + \ell} \quad (12)$$

But... (Harrod's second problem)

- This condition depends on four independent and exogenous parameters:
- s results from the consumption behavior of the households
- μ , λ and ℓ are the properties of the technology of the economy
- and n results from demographic laws.
- Consequently, the compatibility of these behaviors is not *warranted*.
- Moreover, if the economy starts with from a disequilibrium situation (eg. : $\gamma_w < \gamma_n$), no feed-back mechanism can pull it towards the equilibrium, on the contrary!
- “Growth on the edge of the knife”.
- A very pessimistic conclusion on the economic growth and its persistence.
- In contradiction with Fact 5.