Sand in the Wheels or the Wheels in Sand? Tobin Taxes and Market Crashes*

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Abstract

Recent crisis revived interest in financial transaction taxes (FTTs) as a means to offset negative risk externalities. However, up-to-date academic research does not provide sufficient insights into the effects of transaction taxes on financial markets, as the literature has here-to-fore been focused too narrowly on Gaussian variance as a measure of volatility. In this paper we argue that it is imperative to understand the relationship between price jumps, Gaussian variance, and FTTs. While Gaussian variance is not necessarily problem in itself, the non-normality of return distribution caused by price jumps affects not only the performance of many risk-hedging algorithms but directly influences the frequency of catastrophic market events. To study the aforementioned relationship we use an agent-based model of financial markets. Its results show that FTTs may increase the variance while decreasing the impact of price jumps. This result implies that regulators may face a trade-off between overall variance and price jumps when designing optimal tax. However, the results are not robust to the size of the artificial market, as nonlinearities emerge when size of the market is increased.

Keywords: price jumps, financial transaction taxes, agent-based modeling, Monte Carlo, volatility.

JEL Classification Number: C15, C16, C61, G17, G18

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1 Introduction

James Tobin first proposed a tax on spot conversions of one currency into another (Tobin, 1978) in the aftermath of the Bretton-Woods system's break-up as a way to mitigate short-term financial round-trip excursions into another currency. His intention was "to throw some sand in the wheels of our excessively efficient international money markets." He and his co-authors offered more arguments in favor of the tax in Eichengreen et al. (1995). But Tobin's idea was just a specific application of Keynes's idea of a tax on transactions mitigating the effect of speculation on financial markets (Keynes, 2006). However, the name 'Tobin tax' is today often used to denote not only foreign exchange transaction taxes, but financial transaction taxes (FTTs) in general. Therefore, the following text uses these terms interchangeably.

The debate on merits of Tobin-like taxes has not so far reached a definite conclusion. The proponents of the tax claim that increased transaction cost affects short-term high volume trading (speculation) more than long-term positions, decreasing market volatility and thus potential for crashes. In this regard the tax can be thought of as a Pigovian tax on a negative risk externality, as increased volatility can decrease welfare and efficiency. The opponents of the Tobin tax generally claim that it can, in fact, increase volatility by decreasing market liquidity or that speculative trading serves to stabilize prices around long-run equilibrium. Although recently quite widely discussed in policy and political circles, the debate there is often driven by ideology and politics rather than rigorous academic research. The academic debate was historically driven mostly by theoretical models, although more recently simulation and empirical studies have been gaining some ground. However both theoretical predictions and empirical evidence is so far mixed.

Historically, one strand of the literature used macroeconomic models of market bubbles. The arguments against the tax are often based on efficient market hypothesis (EMH from now on; due Fama, 1965), which implies that speculators cannot destabilize market, as they would eventually run out of money and be driven out of the market. Furthermore, based on EMH one can argue that speculative trading provides liquidity and helps to incorporate new information into the prices. Opposing models argue that externalities, imperfect information and other frictions may cause inefficiencies and that in these cases FTTs can help economy reach the second best outcome.

Another strand of literature is focused on microeconomic behavior of the agents of the financial markets.

Earlier examples of heterogeneous agent models include Palley (1999), who combined noise traders (which were shown in prior literature to increase volatility, see e.g. De Long et al., 1990) with the literature analyzing the Tobin tax. He identified conditions under which such a tax drives out noise traders, thus benefiting fundamental traders and lowering volatility and leading to higher efficiency. Also, he concluded that there is a trade-off between costs and benefits, because Tobin tax may discourage fundamental traders, as well. Westerhoff (2003) used model with fundamentalist and chartist traders in a model of foreign exchange markets. In this model, a low tax rate first crowds out chartism, but higher rates lead to misalignment due to decreasing number of fundamentalist. Using a different approach, Mathevet and Steiner (2012) show on a dynamic global game that in the imperfect information setting transaction taxes may stop sudden investment reversals under certain conditions, thus increasing welfare.

The empirical evidence on this issue is scant (one of the reasons is that it has never been adopted in its true form as a global tax) and, as we will argue, methodologically problematic. Few papers that tried to estimate the effect empirically (estimating effect of transaction taxes either on local foreign exchange or financial markets) offer support for all possible sides of the debate. The side that found evidence against the transaction tax includes Umlauf (1993) who, based on time series data on equity returns in Sweden, found that by introducing transaction tax the volatility measured by the conditional variance went up and trading volumes down. Moreover, the author argued that significant amount of trading activity moved to London. However, it must be noted that Swedish transaction tax of 1% (later increased to 2%) was higher[1] than what Tobin proposed originally (0.5%), and the author himself notes that "appropriate theoretical foundations are lacking" making the estimation imprecise and warns against "generalizing from a single data point" (*ibid.* p. 239). Aliber et al. (2003) examined the effect of transaction costs in general on volatility (defined as standard deviation of prices) of foreign exchange rates for four different currencies, and found positive relationship as well. The opposite result, in support of proponents of the Tobin tax, can be found in Liu and Zhu (2009) who found that lowering of transaction costs in Japan lead to higher volatility, implying negative correlation between transaction costs and volatility. Finally, third group of literature have not found any significant effect — see e.g. Hu (1998), who studied effects of stock transaction tax on market volatility and turnover taking advantage of 14 tax changes that occurred in stock markets in Hong Kong, Japan, Korea, and Taiwan

[1]Note although the tax rate was initially 0.5% and later increased to 1%, this tax was nominally borne by both sides of the transaction implying the overall tax rate of 1% and 2%, respectively.

during the period 1975-1994.

We see two major issues that are left rather unexplored. First, scale effect arguably plays a major role in the (Tobin tax was meant to be a global tax). Small markets like Sweden does not have a significant impact on the world economy, so the speculative trading moves abroad, it does not alter the volatility on these foreign markets, but may very much hurt trade volumes domestically. However, if the market is large enough, there will be an impact on foreign market as well. Second, perhaps more importantly, we argue that the studies ignored a significant source of information by focusing on conditional variance as a single measure of volatility. Concerning the first point, some work has been already done. Westerhoff and Dieci (2006) studied the phenomenon in a model with heterogeneous agents who can trade in different markets and can choose a trading strategy (e.g. fundamentalist vs. chartist). Importance of strategies evolve over time according to their fitness. They find that the tax decreases volatility in the market where it was imposed while increasing it on the other. The opposite effect of transaction tax on volatility in the two market framework was obtained by Mannaro et al. (2008), who used the methodology of agent-based models (ABMs). They used four types of traders with different strategies, who can trade on the maximum of two markets. However the relative share of strategies is exogenously input by the authors, but agents may choose where to trade and whether to trade at all. On the other hand, one of the few most recent studies, Bianconi et al. (2009), concluded that transaction tax decreases volatility. Their ABM based on Minority Game framework used again fixed strategies that were randomly distributed across agents at the beginning of the simulation.

Our second – more important and thus far unexplored – point is that all of these studies focused on conditional Gaussian variance as a measure of volatility. They ignore additional source of volatility—price jumps. The literature suggests (Merton, 1976, or Giot et al., 2010) that volatility of most financial instruments can be decomposed into two parts: a regular Gaussian component and a price jump component. Many models aim to estimate conditional variance, such as various GARCH models[2], ignore the price jump component while allowing the realized variance to deviate from the Gaussian distribution. However, as we show in this paper, the link between price jumps and conditional variance is not that straightforward – the measure of one may rise while the measure of the other decreases. Higher conditional variance does not have to be a problem *per se*, because it does not necessarily lead to a leptokurtic return distribution. Fat

[2] For an overview see Hamilton (1994)

tails, which have become a stylized fact of financial markets, are better explained by price jumps, so even if the transaction tax increases conditional variance, its effect on price jump frequency may be opposite, thus making the distribution less fat-tailed. If this is the case, the tax would not only improve the prediction power of standard asset pricing models that use normal distribution but, given that catastrophic events are non-normal in nature, it would lead to higher stability of financial markets. However, the relationship between transaction taxes and price jumps has here-to-fore been rather ignored in the literature.

This paper argues that it is crucial to understand the effect of the Tobin tax on price jumps. As Andersen et al. (2002, 2007) show, price jumps are present in majority of price time series, therefore their presence should be a subject of research. Price jumps can have serious adverse impact on predictive power of the pricing formulas and calculation of the estimates of the financial variables. Moreover, price jumps are the source of non-normality and may cause black-swan events on financial markets.[3]

While the presence of price jumps in the data is well established, the literature disagrees on their origin. One branch of literature (Merton, 1976; Lee and Mykland, 2008 or Lahaye et al., 2011) considers new information a primary source of price jumps, while other authors, like Joulin et al. (2008) and Bouchaud et al. (2006), conclude that price jumps are mainly caused by a local lack of liquidity with news announcements having a negligible effect. The third branch – behavioral finance literature (e.g. Shiller, 2005) – suggests that price jumps are caused by the behavior of market participants themselves. For analyzing the two latter views the ABM methodology is especially appropriate, since it allows for explicit modeling of interactions among market participants.

The principal contribution of this paper is to study the relationship between price jumps and variance, and how transaction taxes affect them. The rest of the paper is organized as follows. We describe the agent based model for simulation of the artificial financial markets in Section 2. In Section 3, we model the impact of the FTT on the price process and provide estimators to quantify this effect. Section 4 discusses results of our analysis. We discuss the importance of the results and avenues for further research in Section 5.

[3] For illustrations of changes in the pricing formulas caused by price jumps see Pan (2002) or Broadie and Jain (2008). Brooks et al. (2011) discuss the effect of higher moments on optimal allocations within utility-based framework.

2 Modeling financial markets with transaction tax

This section introduces the modeling framework to model the financial transaction tax in the financial markets and their impact on the distribution of log-returns with a special focus to extreme price movements. We use the agent-based computational model by Raberto et al. (2003) and Mannaro et al. (2008). Their modeling framework replicates the stylized facts of the financial returns and therefore, in the subsequent part, we implement the recent understanding from the financial econometrics to properly assess the response of the extreme price movements to the FTT.

2.1 Agent-based model

We study the relationship between the price process and the FTT discussed in Section 2 using an agent-based model (ABM). ABMs are especially appropriate for the study of the impact of FTTs on financial markets because:

1. They allow for explicit modeling of said transactions (interactions), not relying on market clearing assumption thus allowing us to study behavior out of steady state;

2. They allow for modeling of each agent independently, and every agent can pick their strategies according to the evolution of the modeled system. This implies that these agents are allowed to be heterogeneous both ex ante and ex post

Our basic treatment follows the methodology of Raberto et al. (2003) and Mannaro et al. (2008) with some modifications. We define four types of agents based on their behavior: random traders, fundamentalist traders, momentum traders, and contrarian traders. We use their values of parameters that were calibrated so that the price series generated match the usual stylized facts of financial markets. The agent-based modeling procedure itself is performed as follows (similar to Lavička et al., 2010): We set initial conditions of the model including number of interacting agents and various model-specific parameters described below. Then we let the economy to evolve step by step until a predetermined number of steps (or trading days) are reached. Every step we record closing price, overall traded amount of assets, amount of assets sold and bought by each trader group, total demand and total supply by each trader group, wealth in each trader group, and tax revenue.

2.1.1 Trader types

Our artificial market consists of traders distributed evenly into four groups based on their decision rules (random, fundamentalist, momentarian, and contrarian). At any given time t, any agent i is characterized by her cash holdings ($c_i(t)$) and asset holdings ($a_i(t)$).

Random traders Random traders denoted as R do not follow any particular strategy, they issue a buy or a sell order with equal probability. They are a proxy for traders that trade for their private reasons independent of the market situation, or who follow irrelevant information. If they buy (sell) the limit price of their buy (sell) order is determined as:

$$l_i^b = p(t) \cdot X, \quad (1)$$

$$l_i^s = \frac{p(t)}{X}, \quad (2)$$

where $X \sim N(\mu, s_i)$. The standard deviation s_i of this Gaussian distribution is determined as:

$$s_i = k \cdot \sigma_i(\omega_i), \quad (3)$$

where $\sigma_i(\omega_i)$ is the standard deviation of the log-returns computed based on window length following uniform distribution $\omega_i \sim U[2,5]$. Parameter k is set to 1.9. As Mannaro et al. (2008) argue, the dependence on past variance simulates a GARCH model trading.

The problem may arise when s_i becomes so large that the realization of $N(\mu, s_i)$ becomes negative. We solve this problem by setting the sell or buy order to zero in these cases.

Fundamentalist traders Fundamentalists trade based on their beliefs about the fundamental price of assets. If fundamentalists decide to buy or sell, they buy/sell a fraction q of their inventory, which depends on current ($p(t)$) and fundamental (p_f) price of the asset:

$$q = k \cdot \frac{|p(t) - p_f|}{p_f}. \tag{4}$$

The parameter k is the same as in the random traders' case. In effect, these traders are arbitrageurs who try to take advantage of differences between market and fundamental price of assets.

Momentum traders Denoted as T, these traders follow trend — they buy when the price goes up and sell when it goes down. They are a proxy for traders using technical analysis or herd behavior. They look back at the history based on a time window ω_i, which is randomly drawn at the beginning of the simulation. If they decide to issue an order, the limit price l_i is computed as:

$$l_i = p(t) \cdot \left[1 + k \cdot \frac{p(t) - p(t - \tau_i)}{\tau_i p(t - \tau_i)}\right], \tag{5}$$

where k is the same parameter as before. In this case $\omega_i \sim U[3, 20]$. Conditional on the decision to sell (buy) the exact quantities are computed as follows:

$$q_i^s = a_i(t) \cdot U \cdot \left[1 + k \cdot \frac{|p(t) - p(t - \tau_i)|}{\tau_i p(t - \tau_i)}\right], \tag{6}$$

$$q_i^b = a_i(t) \cdot U \cdot \left[1 + k \cdot \frac{|p(t) - p(t - \tau_i)|}{\tau_i p(t - \tau_i)}\right], \tag{7}$$

where q_i^s is the quantity to sell and q_i^b is the quantity to buy, and U follows uniform distribution $U(0,1)$. Naturally, agents cannot sell more assets than they possess ($q_i^s \leq a_i(t)$) or spend more cash than they hold ($q_i^b \leq c_i(t)$).

Contrarian traders Similarly to momentum traders, contrarian traders (C) follow technical analysis of the trend, however they expect that if the price is rising it is going to fall soon, so they try to sell near the maximum and vice versa. This implies that their behavioral rules are the same as those of momentum traders, only in the opposite direction.

2.1.2 Tax collection

The tax rate is imposed on both sides of the transaction. More precisely, it is added on top of the price for buyers, and distracted from the sell price for sellers. Thus, the effective tax rate is twice the tax rate in our model. In order not to decrease money supply in our economy, every 60 days we return tax revenues into the system as a lump sum divided among traders while maintaining the existing distribution.

2.1.3 Price clearing mechanism

Market clearing price p^* is determined by an intersection of demand and supply curves. More specifically, the orders are sorted by price: sell orders whose price satisfies $s_v \leq p^*$ from lowest to highest, and buy orders whose price satisfies $b_u \geq p^*$ from highest to lowest. These buy and sell orders are then matched from the bottom of the list while there is at least one pair to be matched. In case the last buy or sell order is satisfied only partially, p^* is determined as a weighted average of the bid and the ask price. Based on this matching, variables a_i and c_i are updated accordingly for each trader who made an exchange.

The provided model thus generates for every trading day a market price along with the volume and other market characteristics describing the profile of each of the four trading groups. In the following, we focus on the price generating process, which by construction, satisfies the standard stylized facts known in the market, see Mannaro et al. (2008); Raberto et al. (2003).

2.2 Model of price process

Throughout this paper, we consider a one-dimensional asset log-price process X that takes the form of the Ito semi-martingale described by the following stochastic differential equation:

$$dX_t = \mu_t dt + \sigma_t dB_t + \int_{\Re} x \cdot \mu\left(dt, dx\right) \,, \tag{8}$$

where $B\left(t\right)$ is a standard Brownian motion, see Jacod and Shiryaev (1987) for an introduction in this field. Such a price model is a suitable and general candidate to model the log-price process in a realistic setup and thus tends to be an appropriate in our Agent-based framework, which gives the price process satisfying the stylized facts.

The spot volatility σ_t is a càdlàg process bounded away from zero almost surely. The drift μ_t is in our case identically equal to zero[4]. Variable $\mu(dt, dx)$ is an integer-valued random measure that captures a jump in X_t over a time interval $[t, t + dt)$. This implies that a jump arrives to the market whenever $\Delta X_t \equiv X_t - X_{t-} \neq 0$. Let us further define a jump intensity $dt \otimes \nu_t(dx)$, where $\nu_t(dx)$ is some non-negative measure with a constraint $\int_{\Re} (x^2 \wedge 1) \nu_t(dx) < \infty$. More precisely, we assume large price jumps with finite activity. As a result, for any fixed interval $[0, T]$ there is a finite number of time moments t such that $\Delta X_t \neq 0$.

For a certain fixed interval $[0, T]$ the jump term with corresponding jump intensity ν_t gives rise to a finite number of price jumps. More precisely, there exists a finite number $t_i \in [0, T]$ such that $U_i \equiv \Delta X_{t_i} > 0$ in the limit, with $i = 1, \ldots, N_T$. In such a case, there are exactly N_T price jumps. The term ν_t thus affects both the U_i, and the grid $\mathcal{T}_T = \{t_{1}, \ldots, t_{N_T}\}$ including its cardinality.

The Tobin tax in the model affects the trading and thus the random processes in Eq.(8). In particular, the process driving the volatility and the jump measure depends on the tax rate τ:

$$\begin{aligned} \sigma_t \rightarrow & \quad \sigma_t(\tau) \\ \nu_t \rightarrow & \quad \nu_t(\tau) \end{aligned} . \tag{9}$$

Estimation of the functional dependence between the spot processes in (9) and the FTT is not a straightforward task, as the randomness in the spot processes would be a confounding factor[5]. Any test would therefore require a comparison of the random processes that depend on the current state of the world. To tackle these issues, we use integrated variables.

Two integrated variables are of particular interest in this context: quadratic variance and integrated variance. For the log-price X_t specified by Eq.(8) defined over a time interval $[0, T]$, we define quadratic variance as:

$$QV_T(\tau) = \int_0^T \sigma_s^2(\tau)\, ds + \sum_{i=1}^{N_T} U_i^2(\tau) \ , \tag{10}$$

where we keep the explicit functional dependence on the Tobin tax. Quadratic variance captures contribu-

[4] The fundamental price in our model is fixed, which is equivalent to a world with zero deterministic interest rates. Alternative and equivalent explanation is that our model describes detrended data.

[5] Recall that σ_t itself is a random process with a structure similar to the log-price equation.

tions both from the spot volatility $\sigma_t(\tau)$ and from the jumps. Integrated variance, on the other hand, is defined as:

$$IV_T(\tau) = \int_0^T \sigma_s^2(\tau)\, ds\,, \tag{11}$$

Thus, integrated variance is defined as an integral over the square of continuous-time spot volatility.

The difference between IV_T and QV_T forms the basis of price jump detection[6]. For this purpose, however, we need consistent estimators of QV_T and IV_T under the log-price process (8). Therefore, let us define realized quadratic variance as:

$$\widehat{RV}_{M,T} = \sum_{i=1}^{M} \left(Y_{i\frac{T}{M}} - Y_{(i-1)\frac{T}{M}} \right)^2 . \tag{12}$$

Then for any sequence of non-stochastic partitions $0 = t_0 < t_1 < \cdots < t_M = T$ that satisfies $\sup(t_i - t_{i-1}) \to 0$ as $M \to \infty$, the log-price process X_t defined over a time interval $[0, T]$ sampled into M equidistant parts exhibits the following property:

$$\widehat{RV}_{M,T} \xrightarrow{p} QV_T\,. \tag{13}$$

Furthermore, let us define realized bipower variance as:

$$\widehat{BV}_{M,T} = \mu_1^{-2} \sum_{i=2}^{M} \left| Y_{i\frac{T}{M}} - Y_{(i-1)\frac{T}{M}} \right| \left| Y_{(i-1)\frac{T}{M}} - Y_{(i-2)\frac{T}{M}} \right| , \tag{14}$$

where $\mu_\alpha = E\left(|z|^\alpha\right)$, with $z \sim N(0,1)$, and $\mu_1 = \sqrt{2/\pi}$. Barndorff-Nielsen and Shephard (2004) showed that for this definition and the same sequence of M partitions as above, realized bipower variance possesses the following property:

$$\widehat{BV}_{M,T} \xrightarrow{p} IV_T\,. \tag{15}$$

Conveniently, the small sample correction coefficient $\frac{M}{M-1}$ is introduced in the definition (14). The literature

[6] See Barndorff-Nielsen and Shephard (2006).

provides a plethora of estimators of integrated variance[7].

2.2.1 Estimating numbers of price jumps

Estimating the price jump contribution to the overall quadratic variance is one way how to assess the role of price jumps in the price process. Alternatively, we may directly identify overall amount of price jumps. For a given sampling frequency, we thus assess the cardinality of the set of returns, which contain at least one price jump.

To test for a presence of a price jump in a particular return, we employ a test developed by Lee and Mykland (2008). As Hanousek et al. (2012) argue, this test is optimal with respect to Type-II errors. It is based on the bipower variance suggested by Barndorff-Nielsen and Shephard (2004) for underlying processes following Eq.(8). The test statistics is based on the results of extreme value theory. More precisely, the key quantity is the distribution of maximum returns normalized by the spot integrated variance. The spot quadratic variance is estimated using the bipower variance over a moving window capturing the immediate past movements of the price process. Namely, the test statistics developed by Lee and Mykland (2008) is defined as:

$$\frac{\max_{t\in A_n}|\mathcal{L}_t| - C_n}{S_n} \to \xi\,, \tag{16}$$

where A_n is the tested region with n observations and $\mathcal{L}_t = r_t/\hat{\hat{\sigma}}_t$, $C_n = \frac{(2\ln n)^{1/2}}{\mu_1} - \frac{\ln\pi + \ln(\ln n)}{2\mu_1(2\ln n)^{1/2}}$, $S_n = \frac{1}{\mu_1(2\ln n)^{1/2}}$, and μ_1, where $\hat{\hat{\sigma}}_t$ stands for the spot bipower variance defined as:

$$\hat{\hat{\sigma}}_t^2 = \frac{1}{T-1}\sum_{u=t-T+1}^{t-1} |r_u|\,|r_{u-1}|\,, \tag{17}$$

where we use different notation from (14) to explicitly stress the moving window u. Note that the term μ_1^{-2} is included in coefficients C_n and S_n.

Lee and Mykland (2008) show that under the null hypothesis of no price jump, the random variable ξ follows the standard Gumbel distribution function $P(\xi \le x) = \exp\left(e^{-x}\right)$. The number of price jumps detected in this way is then counted for a given window, in our case 120 days.

[7]See Dumitru and Urga (2012) for a comprehensive overview.

2.3 Effects of Tobin tax

We study the effect of the Tobin tax on financial markets through changes in quadratic and integrated variance caused by changes in the tax rate. The convergence of estimators in Eqs.(13) and (15) is achieved for every process satisfying the assumptions stated in Eq.(8). We assume that for a reasonable region of the Tobin tax considered in this study, $\tau \in [0, \bar{\tau}]$, these assumptions hold. Therefore, for every tax rate the asymptotic convergence of (13) and (15) takes the form:

$$\forall \tau \in [0, \bar{\tau}] : \begin{array}{ll} \widehat{RV}_{M,T}(\tau) & \xrightarrow{p} QV_T(\tau) \\ \widehat{BV}_{M,T}(\tau) & \xrightarrow{p} IV_T(\tau) \end{array} . \tag{18}$$

Considering well known properties of self-organized systems[8], we do not impose any assumptions on the continuity or smoothness of the spot variance σ_t and the jump process captured by the jump intensity ν_t, which are functions of the Tobin tax. As a consequence, the derivative of $QV_T(\tau)$ and $IV_T(\tau)$—or their respective estimators—with respect to τ may not necessarily exist.

Due to the asymptotics (18) for any fixed $\tau \in [0, \bar{\tau}]$, we analyze the dynamics of contribution of price jumps to the total quadratic variance through the $\widehat{RV}_{M,T}(\tau)$ and $\widehat{BV}_{M,T}(\tau)$, respectively. The variable of interest is the relative contribution of the price jumps to the quadratic variance as a function of τ:

$$\widehat{JS}_{M,T}(\tau) \equiv \frac{\widehat{RV}_{M,T}(\tau) - \widehat{BV}_{M,T}(\tau)}{\widehat{RV}_{M,T}(\tau)} . \tag{19}$$

Then, having estimated $\widehat{RV}_{M,T}(\tau)$ and $\widehat{JS}_{M,T}(\tau)$, we may assess the effect of the Tobin tax on market volatility.

Table 1: Sensitivity of the $\widehat{RV}_{M,T}(\tau)$ and $\widehat{JS}_{M,T}(\tau)$ as a function of τ.

Case	$\frac{\Delta \widehat{JS}_{M,T}(\tau_0)}{\Delta \tau}$	Case	$\frac{\Delta \widehat{RV}_{M,T}(\tau_0)}{\Delta \tau}$
A	$= 0$	D	$= 0$
B	> 0	E	> 0
C	< 0	F	< 0

The sensitivity of the volatility to the Tobin tax can be then expressed through the possible combinations

[8]For example, Lavička et al. (2010) show that such a system may develop phase transitions of the first or second kind.

of effects on $\widehat{RV}_{M,T}(\tau)$ and $\widehat{JS}_{M,T}(\tau)$ that can be seen in Table 1. In particular, the combination (A,D) corresponds to no sensitivity of the volatility process—either diffusion or price jumps—to the Tobin tax. All other combinations suggest some form of sensitivity to the Tobin tax. The combinations (B,F) and (C,E) play a special role. In the first case, the overall realized quadratic variance, usually perceived as market volatility, decreases with the increasing tax rate, while the corresponding contribution of price jumps rises. This means that imposing the Tobin tax leads to a decrease in volatility; however, the volatility is less Gaussian. In the latter case, the mechanism is reversed. As a result, the changing contribution of price jumps to the overall volatility can have serious consequences.

2.4 Simulation procedure

The artificial financial market described above is used for extensive Monte Carlo simulations in a modified Zarja C++ environment for agent-based modeling developed in Lavička (2010)[9]. To evaluate the , we differ market specification for a total set of agents in the economy, relative share of different traders populating the economy, and the probability of trading p described in Section 2.1.1.

Table 2: Simulation parameters

Population	Distribution (R:F:M:C)	p	Population	Distribution (R:F:M:C)	p	Population	Distribution (R:F:M:C)	p	Population	Distribution (R:F:M:C)	p
400	40:30:15:15	0.1	400	40:10:25:25	0.1	10,000	40:10:25:25	0.1	100,000	40:10:25:25	0.1
800	40:30:15:15	0.1	400	40:30:22:8	0.1	10,000	40:30:22:8	0.1	100,000	40:30:22:8	0.1
10,000	40:30:15:15	0.01	400	40:30:8:22	0.1	10,000	40:30:8:22	0.1	100,000	40:30:8:22	0.1
10,000	40:30:15:15	0.05	400	20:30:25:25	0.1	10,000	20:30:25:25	0.1	100,000	20:30:25:25	0.1
10,000	40:30:15:15	0.004	400	20:50:15:15	0.1	10,000	20:50:15:15	0.1	100,000	20:50:15:15	0.1
100,000	40:30:15:15	0.1	400	40:50:05:05	0.1	10,000	40:50:05:05	0.1	100,000	40:50:05:05	0.1
100,000	40:30:15:15	0.05	400	60:10:15:15	0.1	10,000	60:10:15:15	0.1	100,000	60:10:15:15	0.1
100,000	40:30:15:15	0.0004	400	60:30:05:05	0.1	10,000	60:30:05:05	0.1	100,000	60:30:05:05	0.1

For all the specifications above, the initial wealth of agent i both in cash and stocks is set as follows. First the overall cash is divided proportionally among the trader groups. Within the trader groups, the cash is divided following Zipf law. After fixing the tax rate, which remains the same for a given specification, agents begin to interact according to their respective decision rules. Every simulation run is composed of 3600 trading sessions, or trading days, which corresponds to 15 years. First 5 years of market operations are

[9]Downloadable from http://sourceforge.net/projects/politeconomy/.

then considered as an initialization period and those data are not taken into account. Every simulation run is then repeated 200 times for each tax rate. The tax rate is varied from 0% to 3% in 0.05 percentage point increments.

At the end of every trading day of each simulation, we collect following data: market price of the traded asset, daily traded volumes, and behavior and wealth (both in terms of assets and cash) of the different trader types. As a result, for every level of the Tobin tax we obtain 200 samples of 10 trading years worth of daily data. This sample is large enough for robust statistical inference.

To evaluate robustness of the results to changes in parameters, we varied the parameters. Summary of all the specifications can be seen in Table 2.

3 Results

This section covers results of the basic model with 400 traders and a robustness check in which we increased the number of traders to 10,000 to see if there is any nonlinear scale effect that would interact with the effect of the Tobin tax.

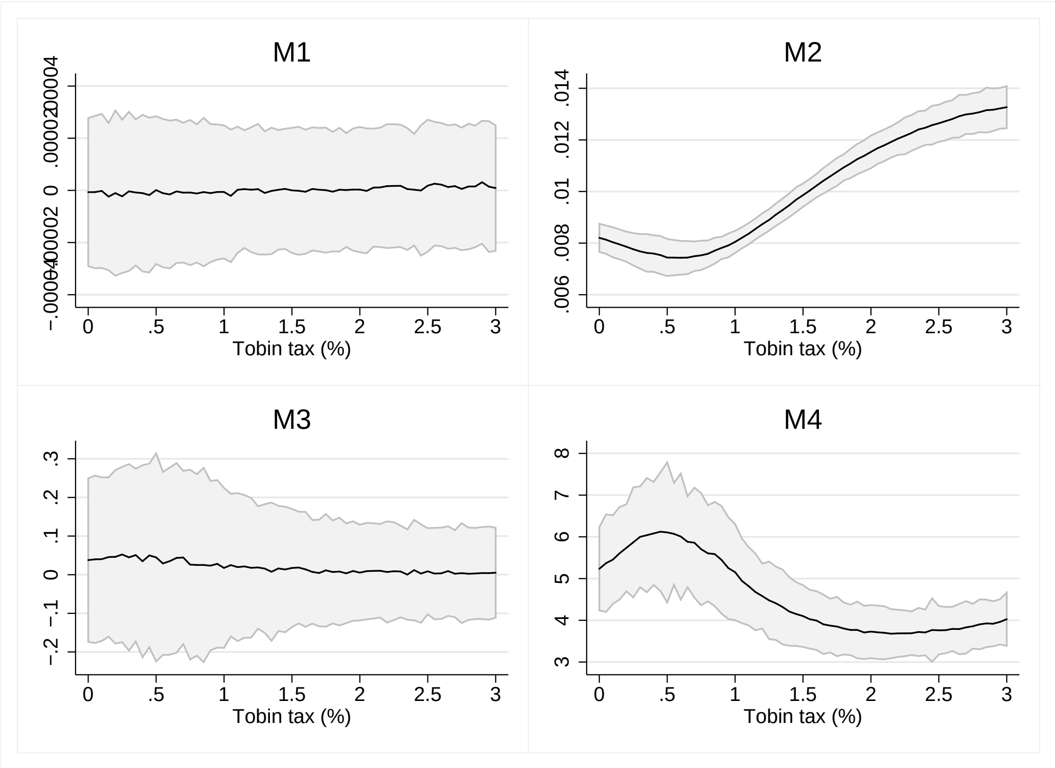
3.1 Market with 400 traders

In this subsection we report results of a simulation with 400 traders, distributed into trader groups described in Section 2.1.1 according to the ratio 40:30:15:15.

3.1.1 Price Behavior

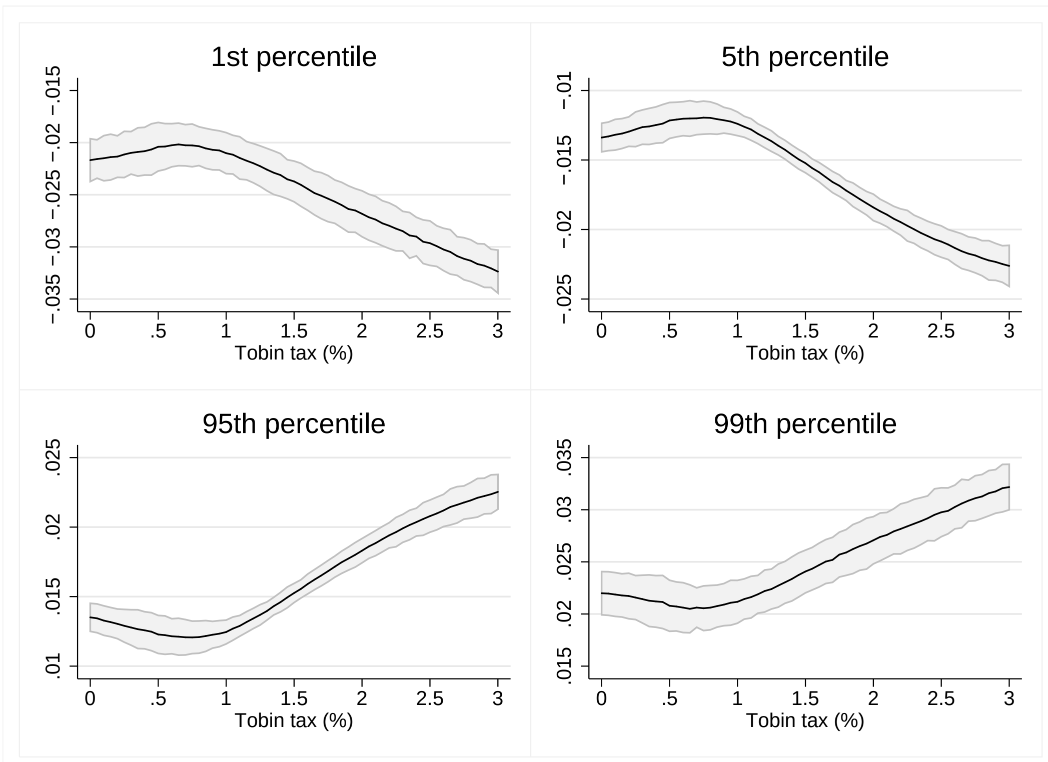
Fig.1 shows first four moments of the distribution of log-returns with 95% confidence bands. It is evident from the figures that the tax has insignificant effect on the mean and skewness of the distribution. Comparison of variance and kurtosis shows that at low levels a rise in the tax has negative effect on variance but increases kurtosis. From approximately 0.5% tax the trend is reversed — variance goes up but kurtosis decreases, making the distribution more Gaussian, albeit with higher standard deviation.

Figure 1: First four moments of the log-return distribution for $N = 400$



Note: The bands represent 95% confidence level from the Monte Carlo simulation.

Figure 2: Centiles for $N = 400$

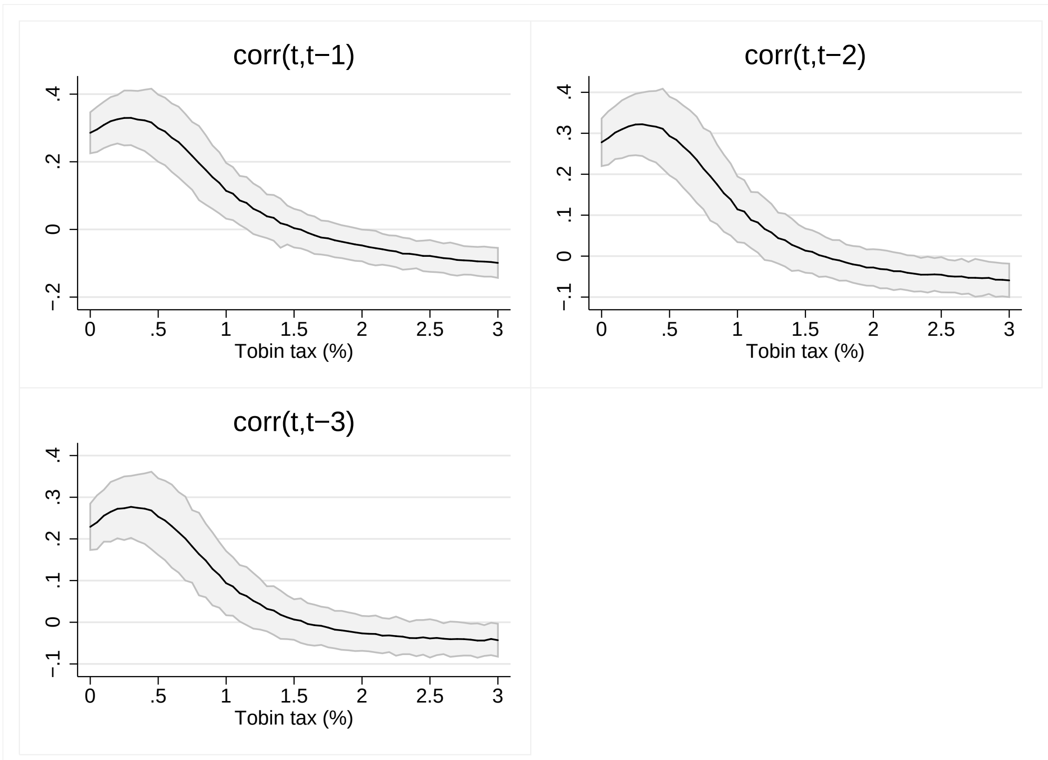


Note: The bands represent 95% confidence level from the Monte Carlo simulation.

3.1.2 Memory

Memory of the log-return generating process expressed as AR coefficients $Corr\left(r_t, r_{t-1}\right)$, $Corr\left(r_t, r_{t-2}\right)$, and $Corr\left(r_t, r_{t-3}\right)$ respectively can be seen in Fig.3. The pattern repeats itself — the memory increases at first, however at higher levels of the tax it decreases.

Figure 3: Memory of the log-return process for $N = 400$



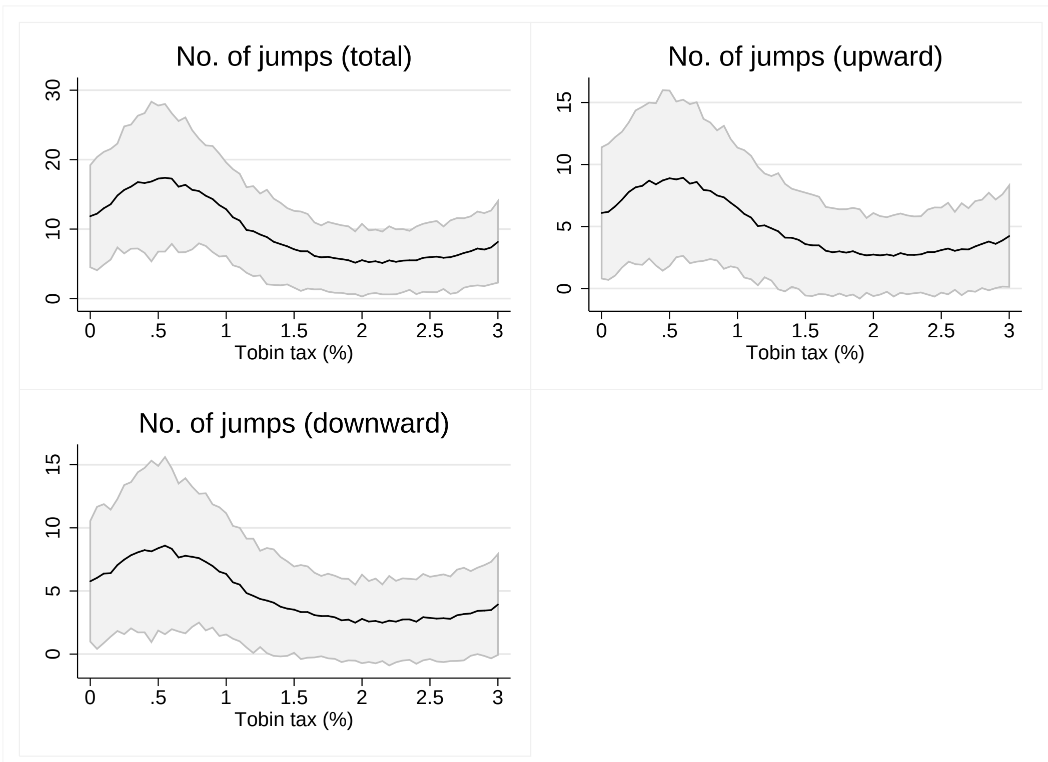
Note: The bands represent 95% confidence level from the Monte Carlo simulation.

3.1.3 Jump statistics

In the following text, we explore the rate of price jump arrivals in greater detail. We employ the test statistics in (16) with 95% confidence interval and identify price jumps in the entire sample for each tax rate. The size of sample is 3000 trading years, which gives 720,000 observations of daily prices. In addition to overall price jumps, we also study upward and downward jumps separately.

Figure 4 depicts the number of identified price jumps as a function of the tax rate. The rate of overall, upward and downward price jump arrivals increases with increasing tax rate at first, and this increase reaches maximum at 0.5% tax. In conclusion, in order to decrease the number of price jumps, the tax rate in the model has to be higher than approximately 1%. This intuition is in line with the pattern exhibited by variance and kurtosis in Fig.1.

Figure 4: Number of jumps for $N = 400$



Note: The bands represent 95% confidence level from the Monte Carlo simulation.

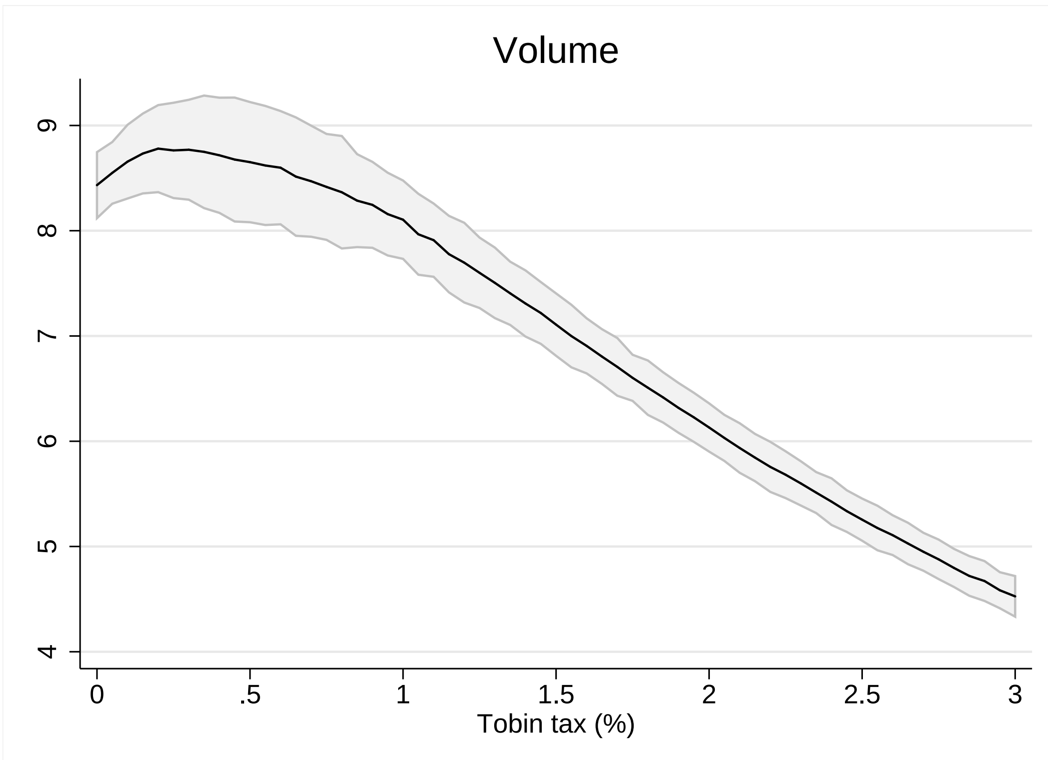
3.1.4 Market Microstructure

3.1.5 Aggregate market data

First, we focus on the analysis of the price time series and traded volumes as a function of the Tobin tax, as decrease in liquidity is allegedly one of the main costs of FTTs.

Fig.5 depicts the relationship between traded volumes and the tax rate. The results clearly show that traded volume is not a monotonic function of the tax rate but rather is maximized around the tax rate of 0.15%, which corresponds to 0.3% overall tax rate. Let us turn our attention to Figure 6, where we analyze the response of the supply and demand to the imposed tax rate. Both demand and supply are monotonically decreasing with the tax rate, meaning the slight concavity of the volume function is caused by ask and bid price not being matched.

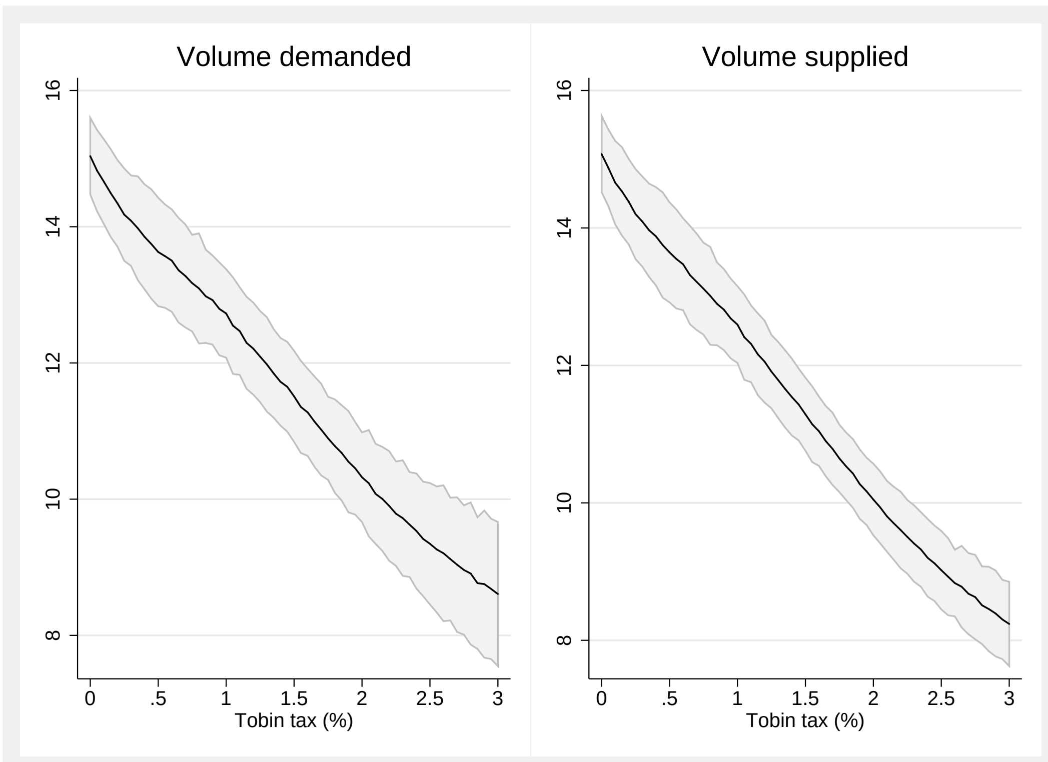
Figure 5: Average trading volume for $N = 400$



Note: The bands represent 95% confidence level from the Monte Carlo simulation.

The liquidity is, however, only one argument in the Tobin tax debate, so let us now analyze returns to better understand the effect of FTTs on risk. Graphs of the first four central moments of the daily log-returns in Figure 1 show two important results. First, standard deviation of returns goes up (which is similar to the results of Mannaro et al., 2008). Second, and more importantly, skewness and kurtosis decrease in absolute value, making the distribution more "normal". While there is no significant correlation between mean of

Figure 6: Average supplied and demanded volumes for $N = 400$



Note: The bands represent 95% confidence level from the Monte Carlo simulation.

returns and tax rate. The increase in normality is also supported by weekly and monthly returns (results available upon request).

The basic analysis of log-returns therefore suggests that the deviation of the price process from normality is decreasing with the increasing tax rate. Standard deviation of returns goes up, which corresponds to Case E in Table 1.

3.1.6 Market microstructure

To determine what exactly drives these results we now turn our attention to the market micro-structure of our artificial market. More precisely, we focus on changes in the aggregate behavior of the four trading groups caused by the variation in the tax rate. Figure 7 reports the average daily inventories— assets and cash—for the four groups as a function of the Tobin tax. For random and contrarian traders, an increase in the tax rate has a negative effect on both asset and money stocks. Trendists' wealth and inventories exhibit local maximum at around 0.5% tax rate. Finally, fundamentalist traders benefit from the growing Tobin tax. The amount of money and assets they hold is positively affected by the tax rate. This evidence suggests that Tobin tax rate affects fundamentalists' and other traders' asset stocks in the opposite way.

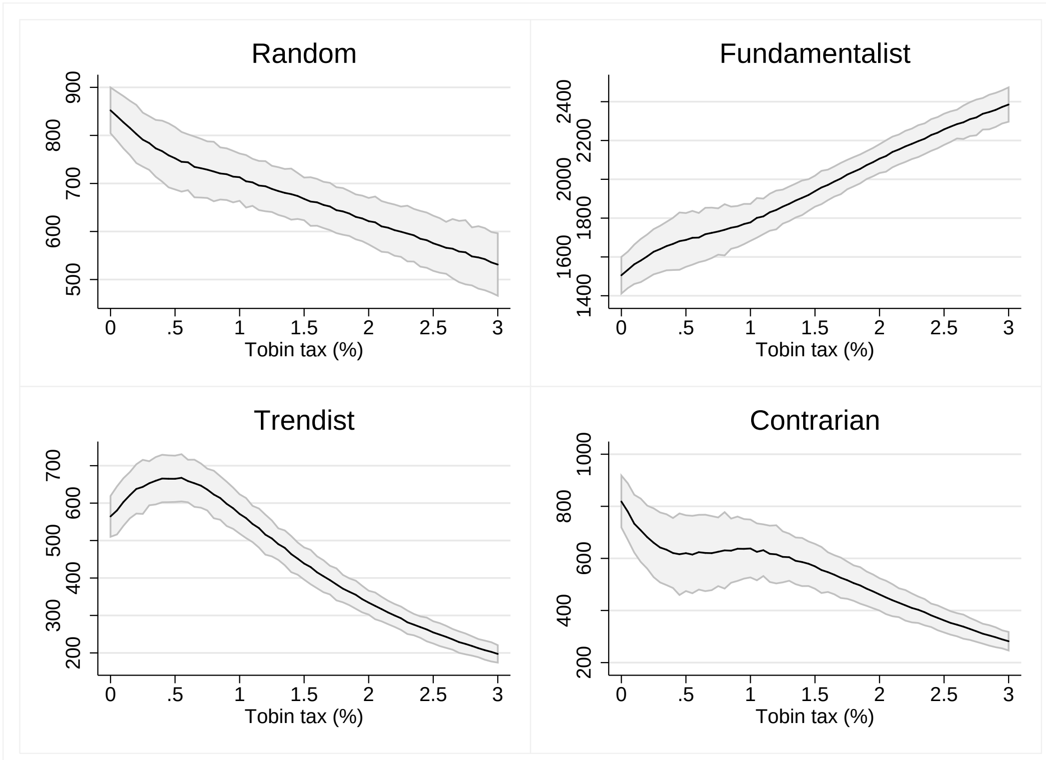
The effect of the tax on the price process and the rate of price jumps is directly connected to liquidity of the market. Figure 8 reports the daily averages of supply and demand of the assets by the respective trader groups. Both demand and supply exhibit similar patterns. While random traders' quantities decrease almost linearly with increases in the tax rate, the response of other trader groups is not monotonic. Fundamentalists and trendists demand more with higher tax rate up to approximately 0.4% and then their activity decreases (although fundamentalists' supply and demand starts to go up again near 2%). Contrarians response to the tax, although almost monotonically negative, is not linear.

Figure 9 shows us the results of the interaction of supply and demand. It reports the average amount of assets sold and purchased by individual traders. The pattern of response to the imposed Tobin tax is similar to supply and demand for all groups except contrarians, which exhibit a hump-shaped relationship, maximized around 1.2%.

Since fundamentalists' demand for assets is relatively less affected by the tax (compared to random and trendist traders) and contrarian traders even increase activity up to a certain level of the tax rate, the

Figure 7: Inventories by traders for $N = 400$

(a) Assets



(b) Cash

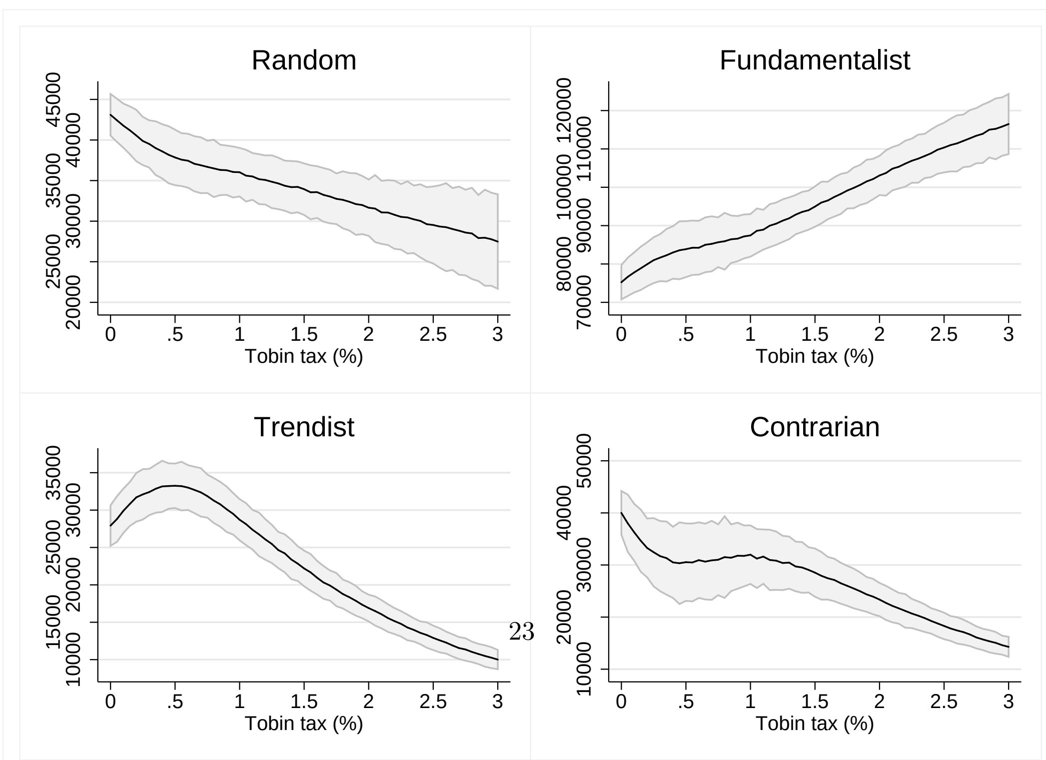
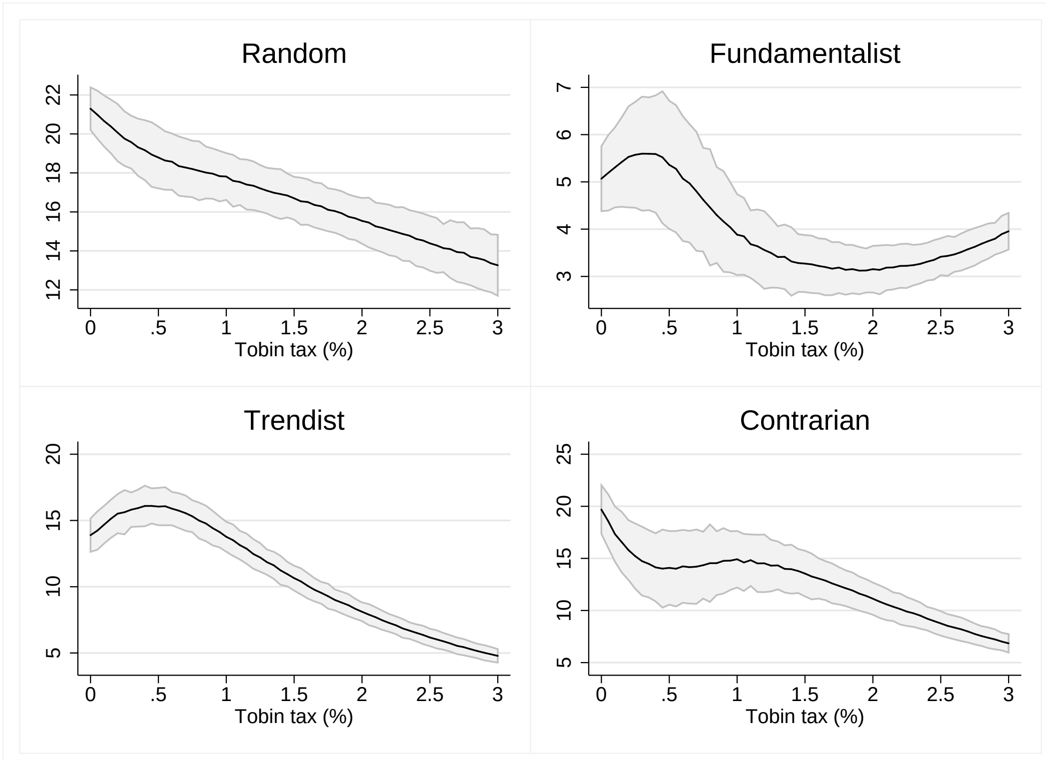
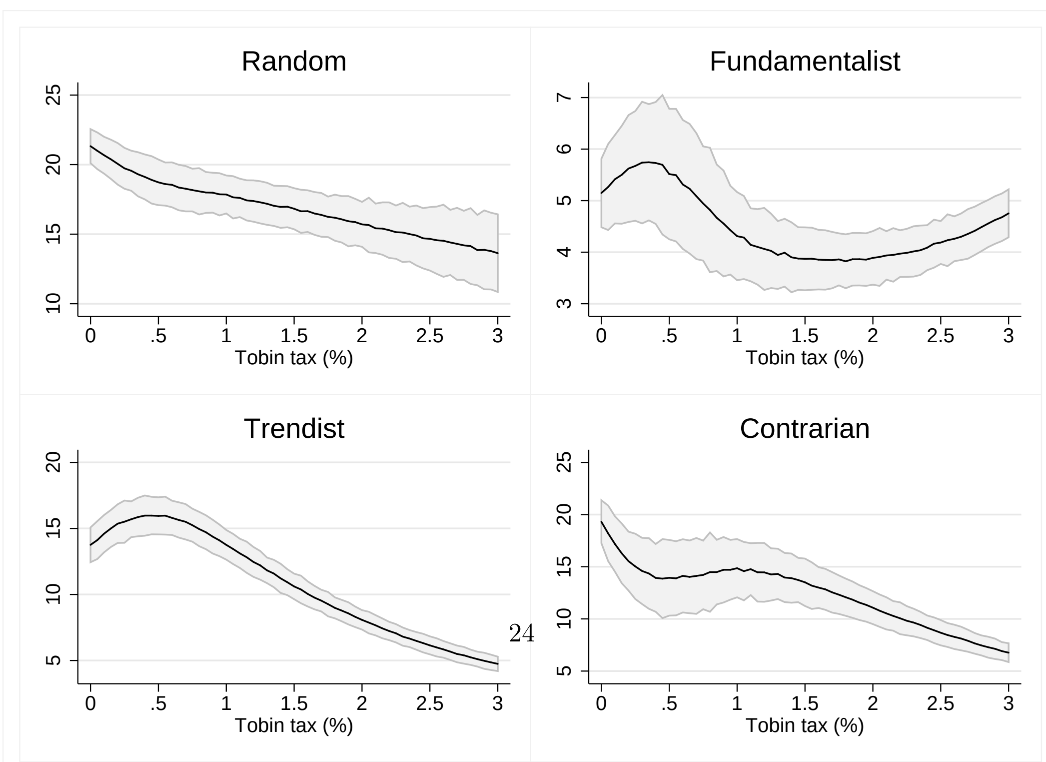


Figure 8: Market order book by traders for $N = 400$

(a) Supply



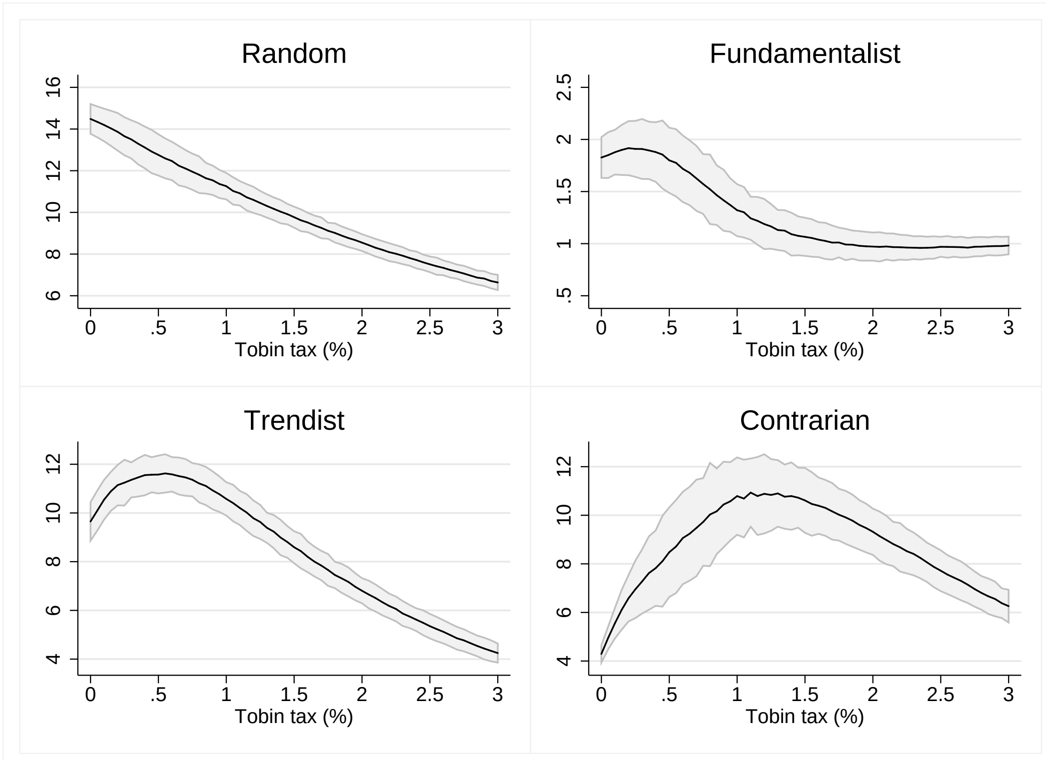
(b) Demand



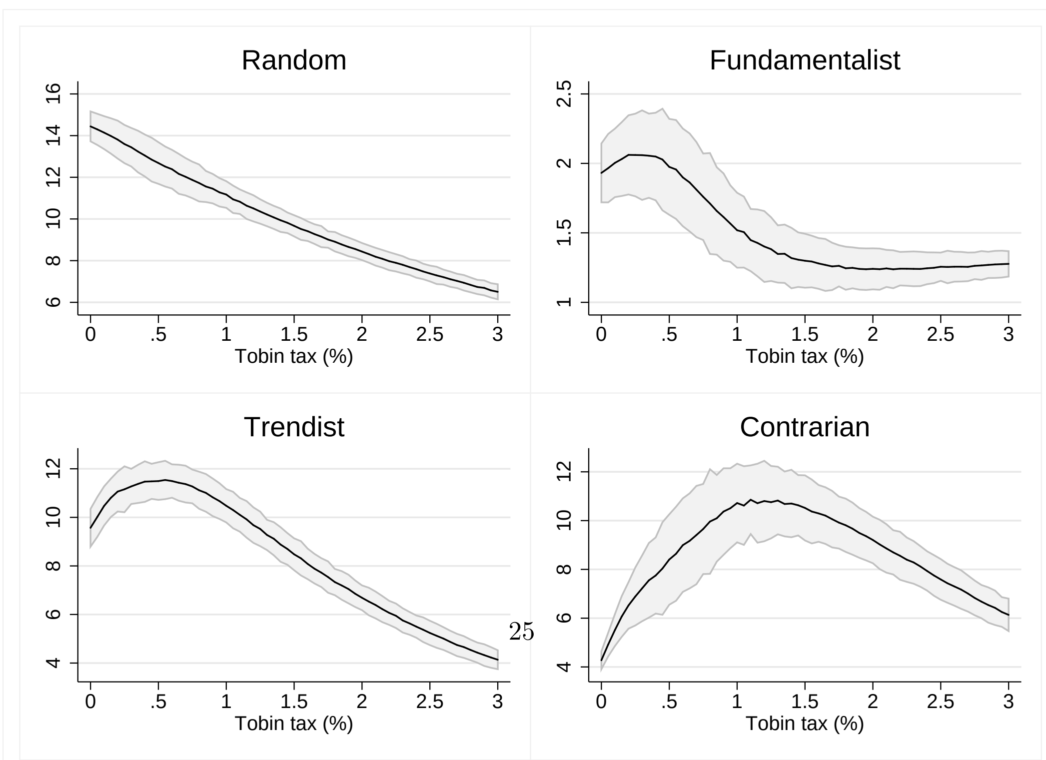
Note: The bands represent 95% confidence level from the Monte Carlo simulation.

Figure 9: Market activity by traders for $N = 400$

(a) Sold assets



(b) Purchased assets



Note: The bands represent 95% confidence level from the Monte Carlo simulation.

results show that the activity of these two groups is (seemingly) explaining the decrease in the number of price jumps. This result seems analogous to previous literature, where fundamentalists served a stabilizing role in the model, although this is the first time it has been shown in the context of price jumps, as opposed to Gaussian variance.

3.2 Market with 10,000 traders

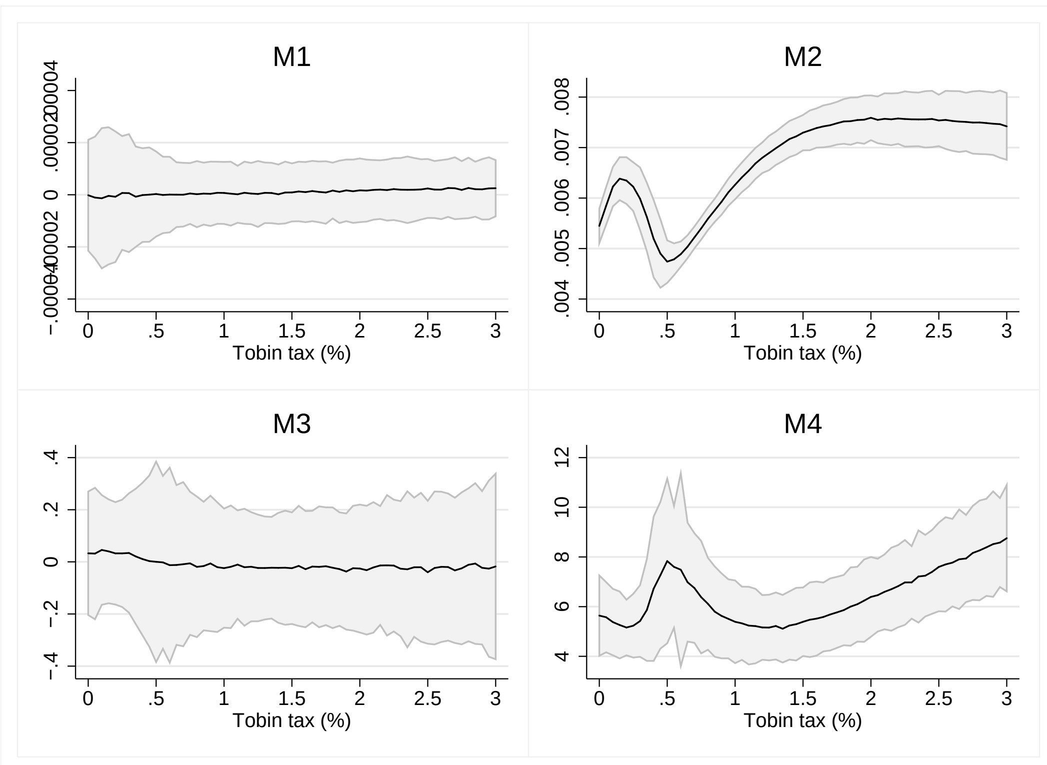
To determine the effect of the market size on the results, we now report the results of a market simulation containing 10,000 traders. Their composition is the same as in the previous case. It is clear from the following figures that there is a significant increase in the size of nonlinearities. Especially second and fourth moments in Fig.10 exhibit a more pronounced spikes at 0.5% than they do on the smaller market. Number of jumps (Fig. 13) exhibits a kink around 0.5%, and overall it goes up, rather than down, with increasing tax rate. The asset and cash holdings (Fig.16), demand and supply (Fig.17), and market activity (Fig.18) show a pattern similar to the one at the smaller market, only with more pronounced nonlinearities.

3.3 Radars

Figure 19 depicts the amount of assets per trader held by every type of the trading strategy in the model. The lines are sorted according to the shades of grey where the black one corresponds to zero tax rate and lighter shades imply higher tax rates. The steps are 0.5%. As we keep strategies fixed, the amount of assets held by every trader can be perceived as attractivity of the given trading strategy. The relative attractivity of different strategies can be interpreted as a likelihood of changing the strategy, if possible, towards the strategies with higher attractivity.

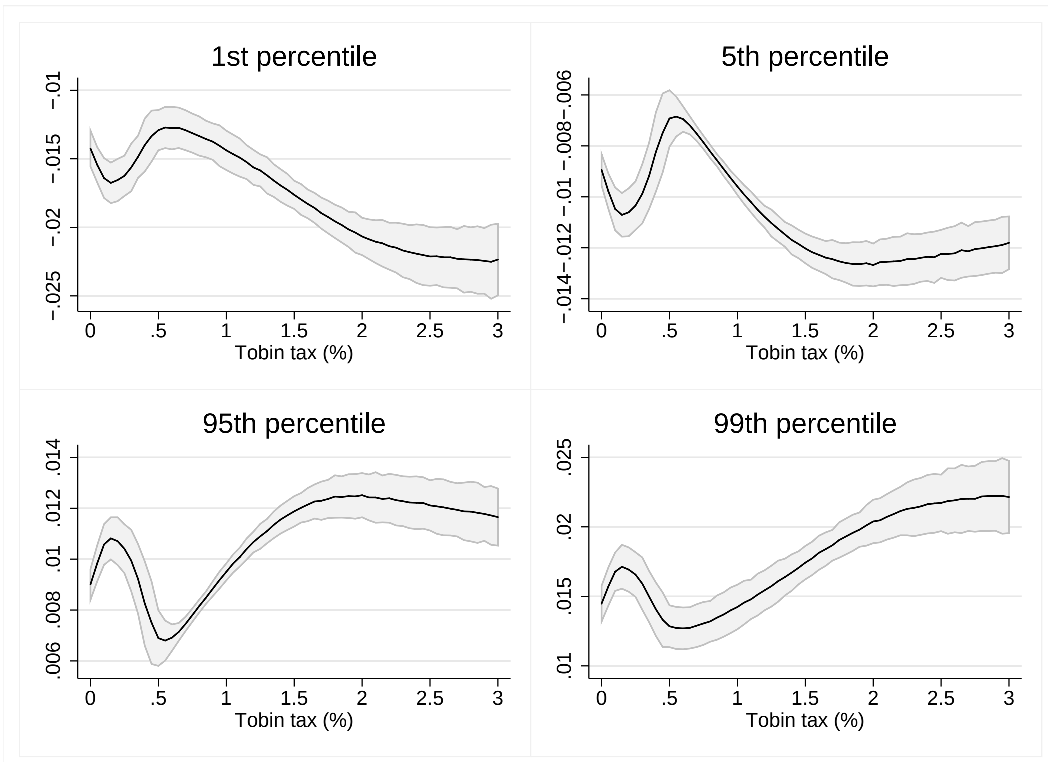
The figure clearly shows that growing tax rates would make the fundamentalist strategy more attractive if the traders could change to it. The other strategies tend to be decreasing in popularity as the Tobin tax rises, though the trend is not straightly monotonous, as we can see for 0.5% tax level and the Trend follower. However, the Trend follower strategy is being suppressed as the rate of the tax rises. In addition, for zero tax rate we see that the Trend follower is the strategy with the least amount of held assets. This suggests that the share of traders with this strategy would be decreasing in a dynamic setting. As a consequence, the fact that we hold the amount of trend followers about the level suggested by the above mentioned attractiveness,

Figure 10: First four moments of the log-return distribution for $N = 10,000$



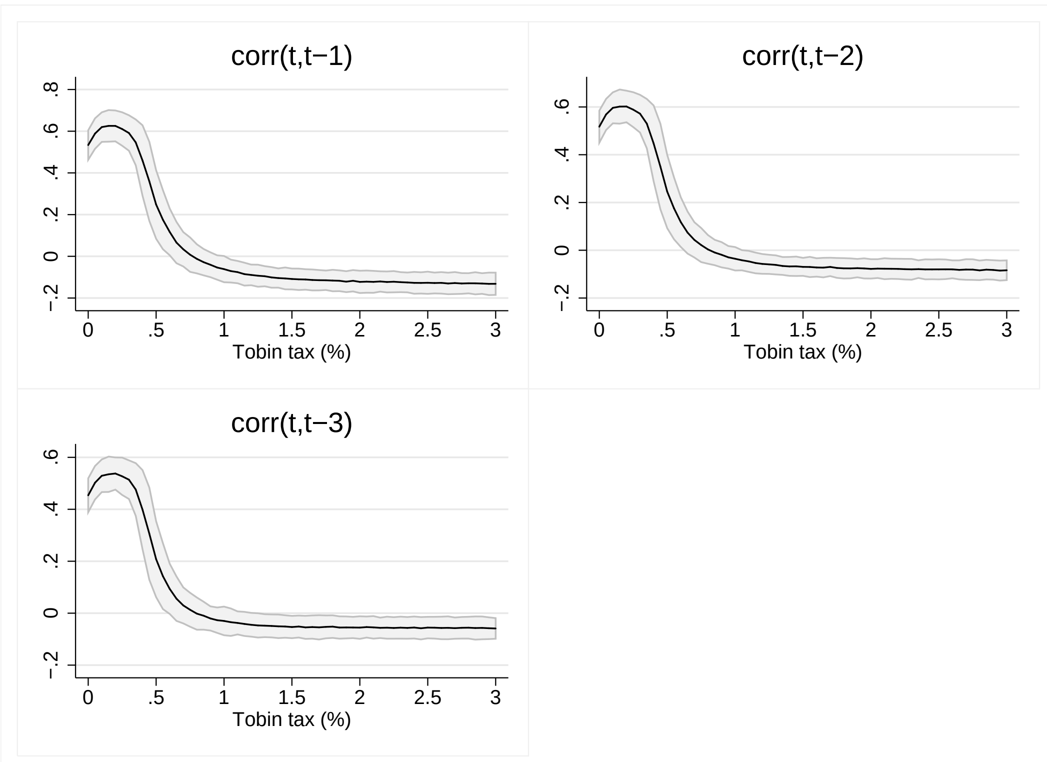
Note: The bands represent 95% confidence level from the Monte Carlo simulation.

Figure 11: Centiles for $N = 10,000$



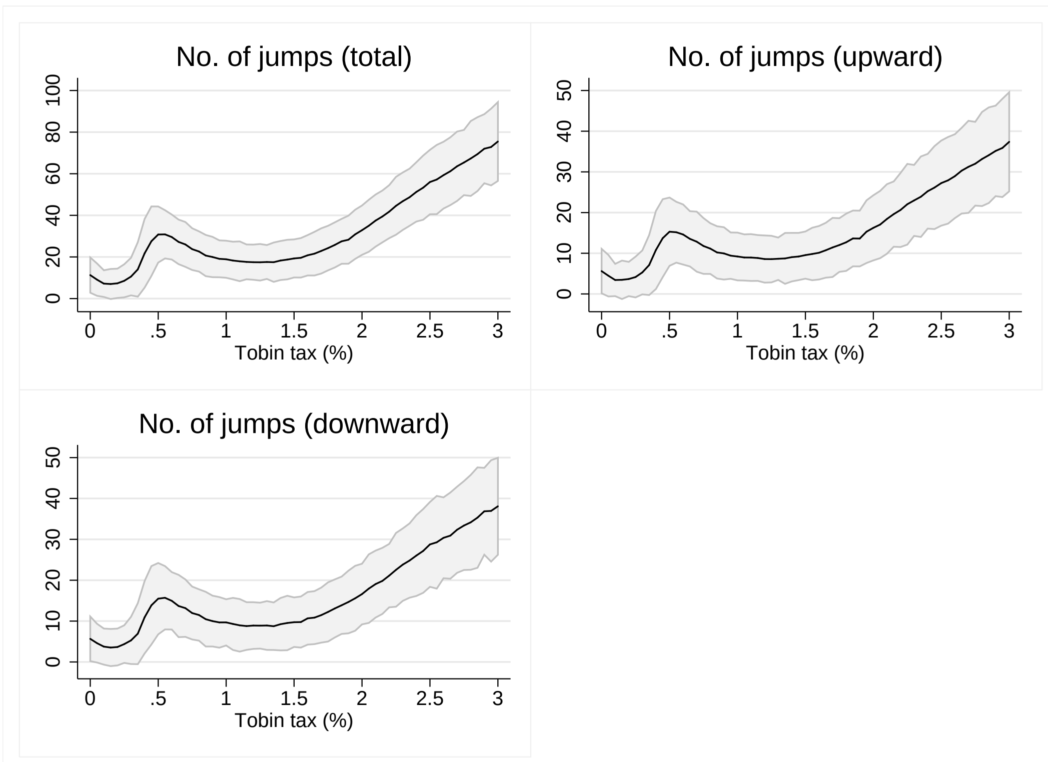
Note: The bands represent 95% confidence level from the Monte Carlo simulation.

Figure 12: Memory of the log-return process for $N = 10,000$



Note: The bands represent 95% confidence level from the Monte Carlo simulation.

Figure 13: Number of jumps for $N = 10,000$



Note: The bands represent 95% confidence level from the Monte Carlo simulation.

Figure 14: Average trading volume for $N = 10,000$

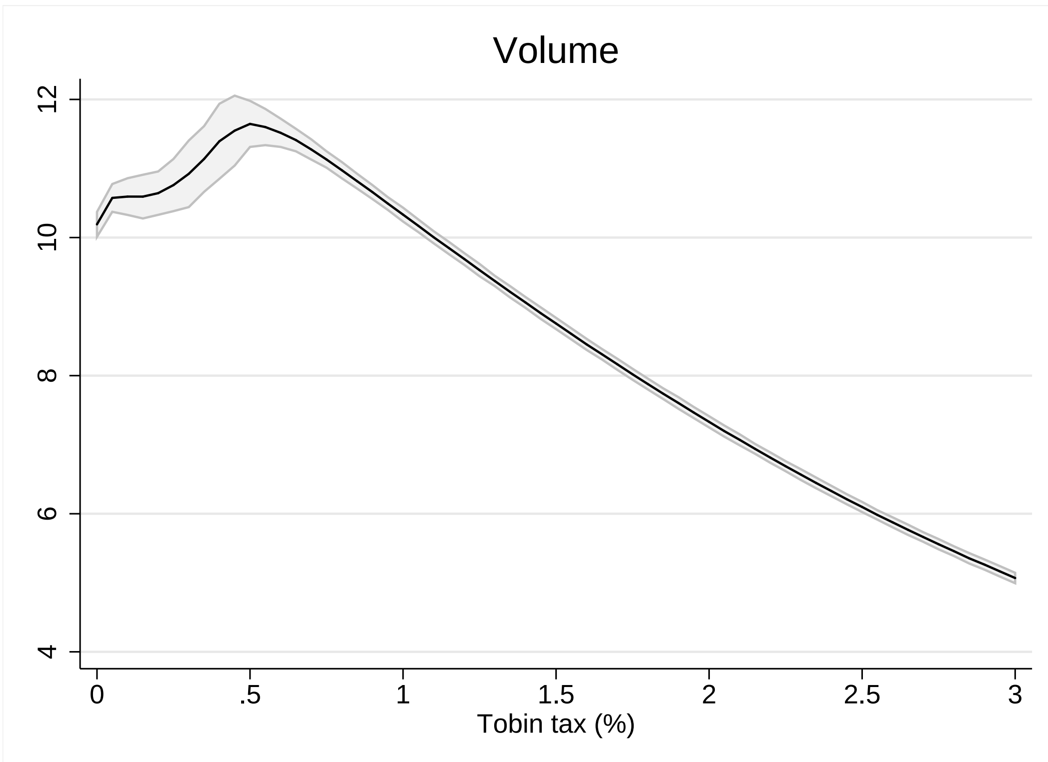


Figure 15: Average supplied and demanded volumes for $N = 10,000$

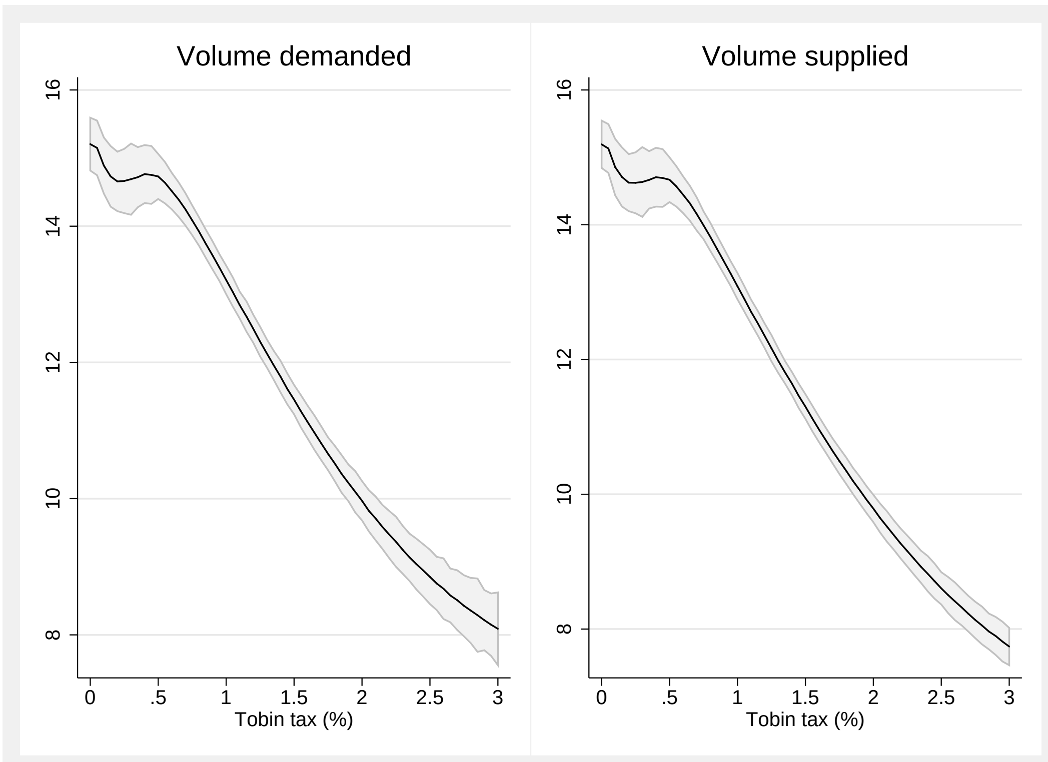
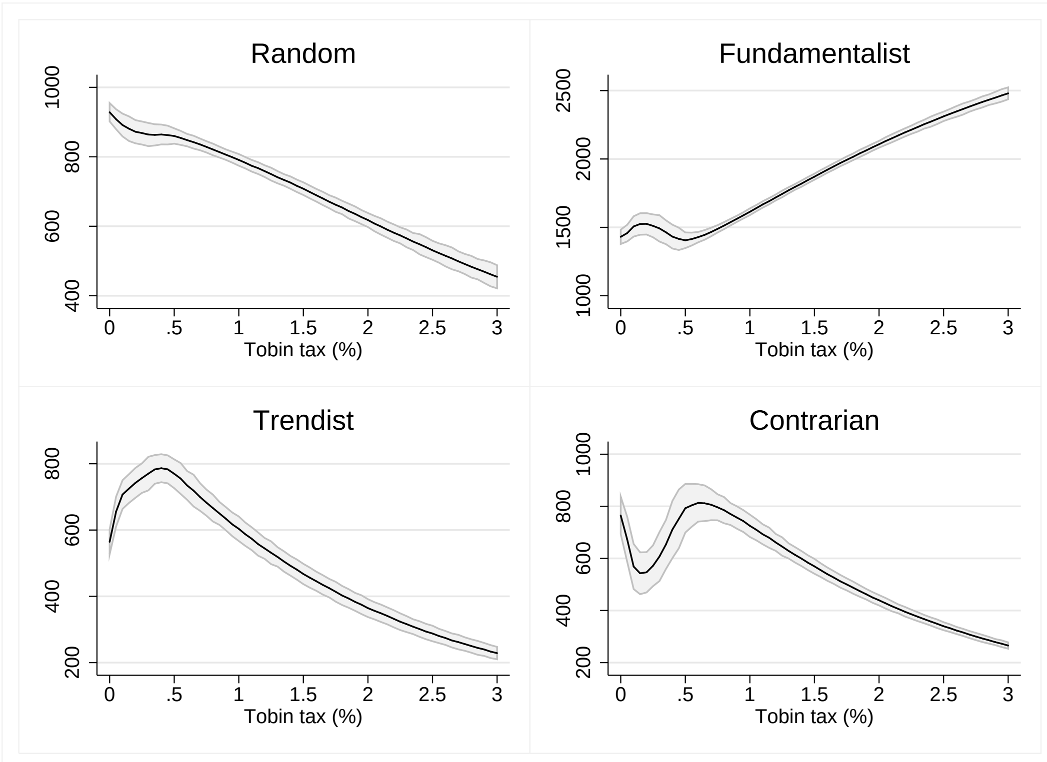
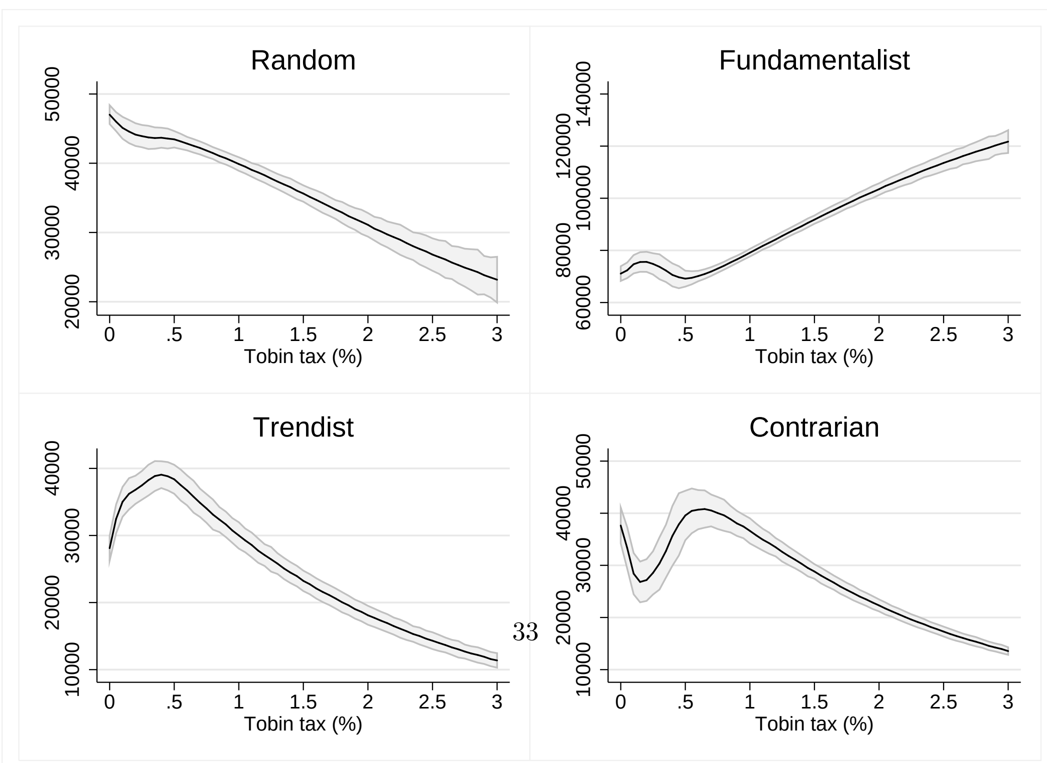


Figure 16: Inventories by traders for $N = 10,000$

(a) Assets



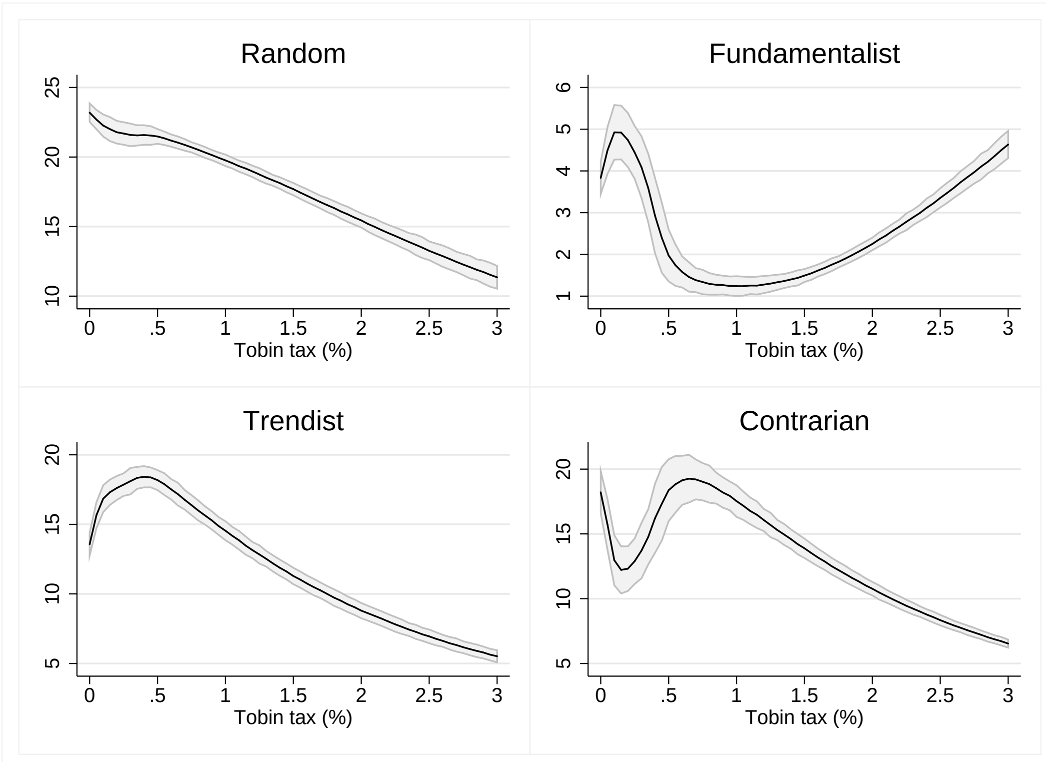
(b) Cash



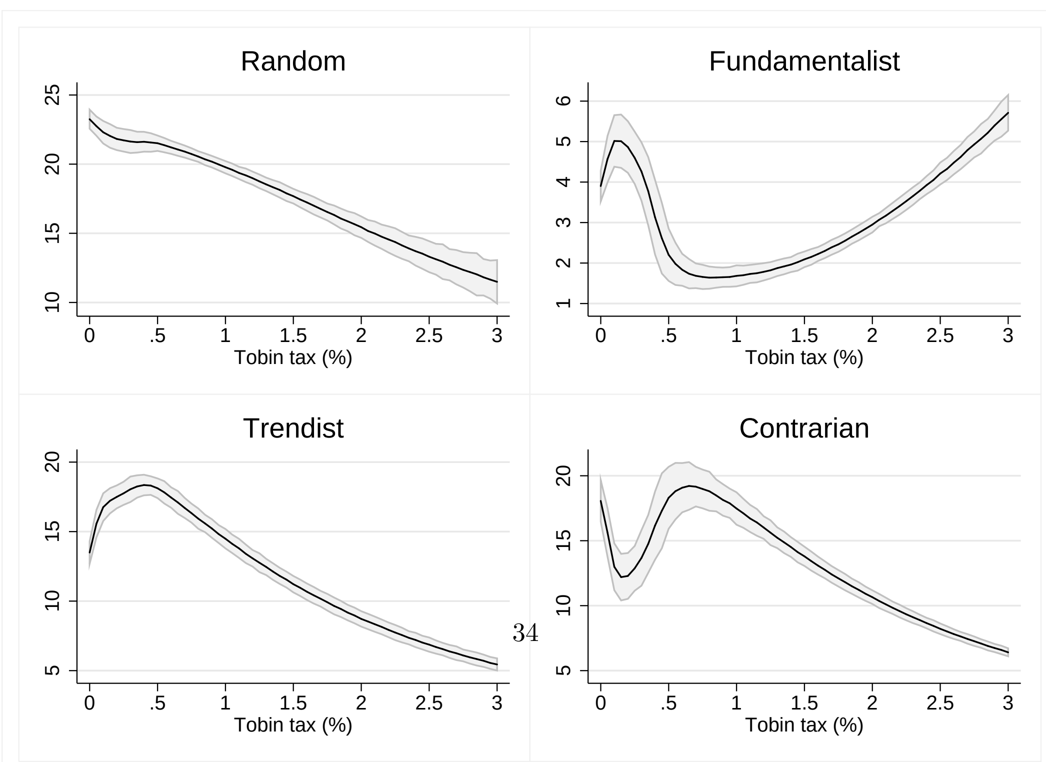
Note: The bands represent 95% confidence level from the Monte Carlo simulation.

Figure 17: Market order book by traders for $N = 10,000$

(a) Supply



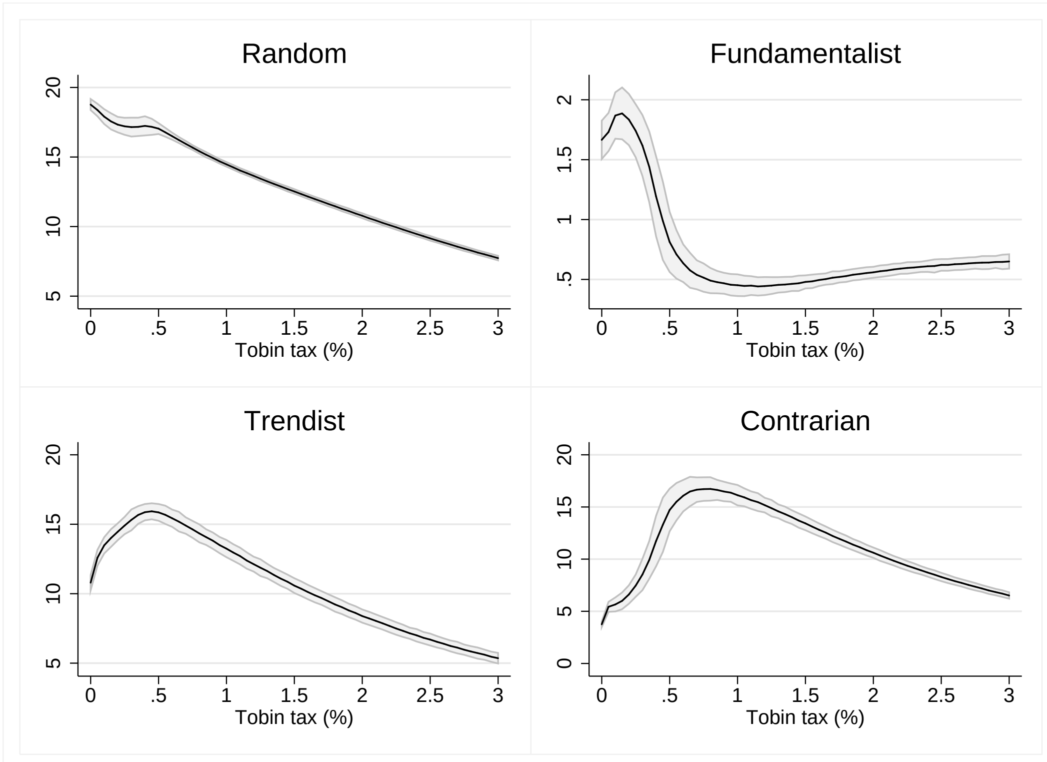
(b) Demand



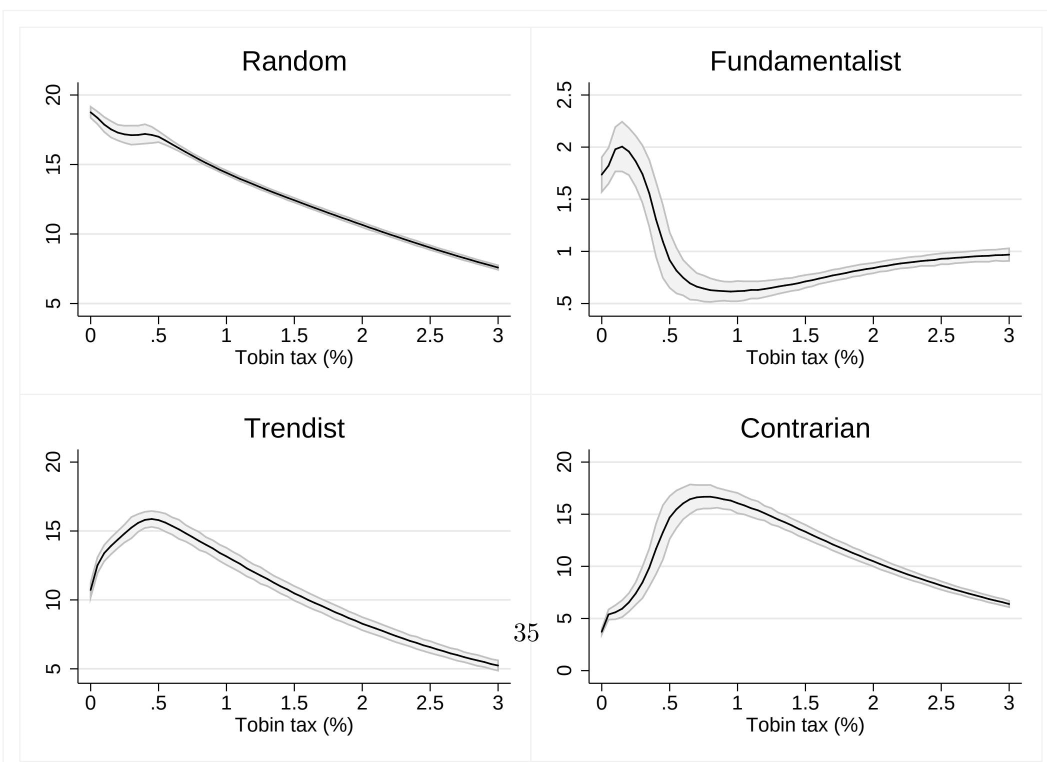
Note: The bands represent 95% confidence level from the Monte Carlo simulation.

Figure 18: Market activity by traders for $N = 10,000$

(a) Sold assets



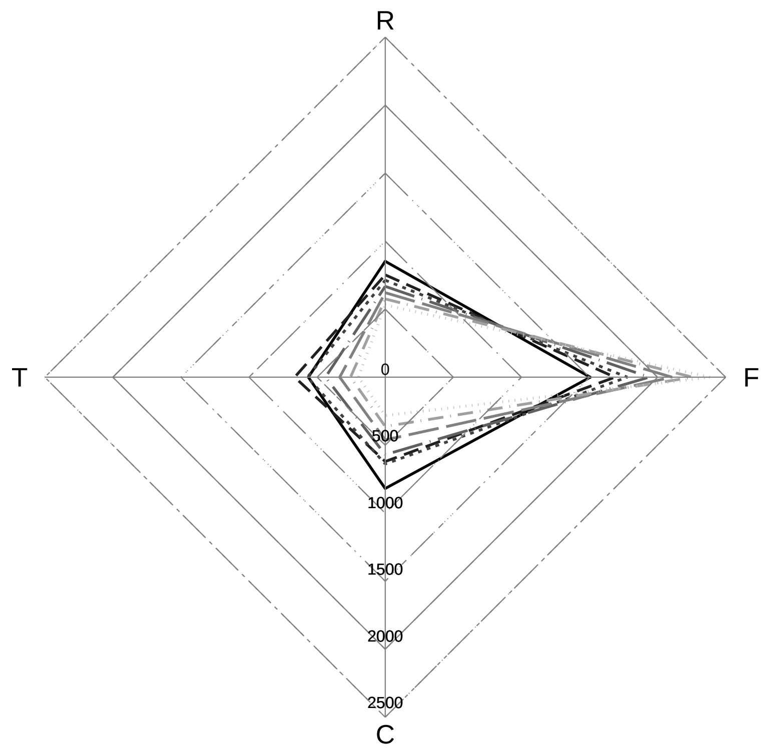
(b) Purchased assets



Note: The bands represent 95% confidence level from the Monte Carlo simulation.

we may introduce the memory into the system. Namely, the trend followers are a type of a strategy which in reality corresponds to bubble builders, i.e., traders, who heat up the bubble-in-progress. The fixed share of strategies does not allow a market correction against this phenomenon, hence the memory in Figures 3 and 12. This implies that the decrease in auto-correlation coefficients is caused by the diminishing activity of trend followers.

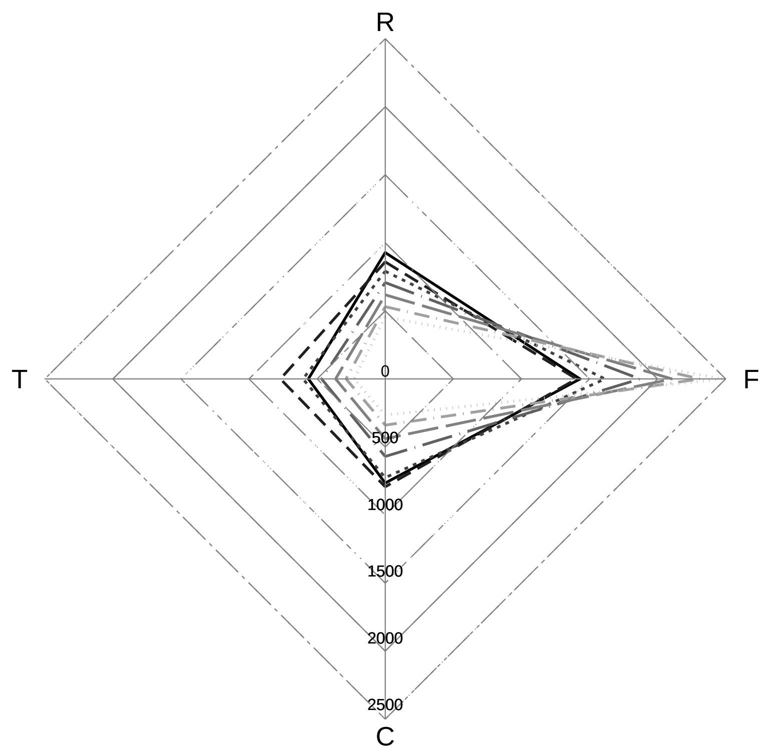
Figure 19: Assets by trader' types for $N = 400$.



Note: The lines are ranked according to shades of grey where the black one corresponds to zero tax rate. The step is 0.5%.

As we see from Figure 20, the story for the larger market is similar.

Figure 20: Assets by trader' types for $N = 10,000$.



Note: The lines are sorted according to the shades of grey where the black one corresponds to Tobin tax equal to 0 and the lighter the line is the higher the tobin tax is. The step are by 0.5%.

4 Conclusion

The main goal of this paper was to open discussion on the here-to-fore ignored relationship between financial transaction taxes and price jumps. We argued that looking at the effect of FTTs on realized variance as a measure of volatility is insufficient, as it does not convey enough information. Our point was that an increase in variance itself does not necessarily mean less stable markets, because realized variance can be decomposed into two parts – Gaussian variance and price jumps. As we have shown, the variance may go up through an increase in Gaussian variance, while the contribution of price jumps may go down, decreasing the kurtosis of the return distribution. This result seems to be driven by different responses of individual trader types to the tax. More precisely, relative weight of fundamentalists in our model is an increasing function of the tax rate. Given that there is a sizeable literature on hedging against Gaussian variance, this result implies that such a tax may improve efficiency of these formulas, and through that, functioning of markets. We believe that our work opens up interesting avenues for further research on the relationship between FTTs and price jumps. More precisely we aim to apply the model in the empirical setup and test the hypotheses implied by this model using the asset prices data. Moreover, as Mannaro et al. (2008) themselves argue, more a model with more sophisticated, possibly risk averse, agents may provide different results, serving as a robustness check.

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